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A Procedure for Quasi-Equiripple Linear-Phase IIR Filters Design

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***Abstract*—The linear-phase IIR filters are described in many cases, mainly due to distortion-free transmission of signals. One of the major problems of IIR filter design is stability, which can be obtained with suitable value of group delay τ. This paper concerns calculation of filter order N and group delay τ in case of quasi-equiripple design of IIR filters. We propose a novel procedure for determining N and τ values; the procedure is valid for all types of filters with arbitrary number of zeros and a few non-zero poles. Evaluation of the proposed approach as well as examples illustrating its application are provided in the paper.**

***Keywords*—Digital filters, IIR filters, filters design.**

I. INTRODUCTION

INFINITE IMPULSE RESPONSE (IIR) digital filters, which approximate both magnitude and phase response, are considered in many papers, for example, for linear-phase filters design. Generally, there are various approaches to design stable linear-phase IIR filters. Such filters can be achieved by:

1) implementation of a phase equalizing allpass filter cascaded with nonlinear-phase IIR filter [1],
2) model-reduction techniques which are applied to approximate the frequency response of finite impulse response (FIR) filter [2], [3],
3) a direct way, i.e., the cost function of the design optimization problem is directly based on desired frequency response [4], [5], [6], [7], [8], [9], [10], [11], [12], [13].

In this paper we focus on the latter method. The transfer function $H(z) = B(z)/A(z)$ of the IIR filter, designed in a direct way, can be obtained by minimization of the following cost function:

$$E_0 = \left(\sum_{i=0}^{L-1} W_0(\omega_i) \left| H(e^{j\omega_i}) - D(\omega_i) \right|^p \right)^{\frac{1}{p}}, \qquad (1)$$

where: $D(\omega)$ is the desired complex-valued frequency response, $W_0(\omega)$ is a real non-negative weighting function, and $i = 2\pi i/L \;\; (i = 0, 1, ..., L-1)$. The case $p = 2$ is called least squares approximation, and the case $p = \infty$ is called complex Chebyshev or minimax approximation. The minimization of E_0 leads to a nonlinear optimization problem which can be solved by use of the Gauss-Newton method or by solving linear equations iteratively when the error (1) is replaced with:

$$E^{(k)} = \left(\sum_{i=0}^{L-1} W^{(k)}(\omega_i) \left| B^{(k)}(e^{j\omega_i}) - D(\omega_i) A^{(k)}(e^{j\omega_i}) \right|^p \right)^{\frac{1}{p}}. \qquad (2)$$

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The new weighting function $W^{(k)}(\omega_i)$ is updated in each iteration k, as follows:

$$W^{(k)}(\omega_i) = \frac{W_0(\omega_i)}{|A^{(k-1)}(e^{j\omega_i})|^p}. \qquad (3)$$

It should be noted that quasi-equiripple approximation of $D(\omega)$ can be also achieved for $p = 2$ by minimization of E. For this purpose it is necessary to add another iteration process that updates the weighting function $W_0(\omega)$ [7] (Lawson-type algorithm) or transforms the desired frequency response [11]. The IIR filter design procedures have to guarantee the filter stability which is generally non-easy task. There exist several ways of ensuring the stability of filter resulting from optimization of (1), with good survey of such methods having been presented in [12]. The optimization techniques used for stable IIR filter design can be of either constrained or unconstrained type. The latter include the concept of prior setting of the group delay value corresponding to the desired frequency response of the filter - confer, e.g. [13] - unfortunately with no explicit formula for calculating this parameter. The former - constrained optimization related techniques - include methods involving positive realness stability domain [10], Rouche's theorem [6], and methods using argument principle for establishing stability criterion [5], [4], [9]. However, also for these methods the filter order should be properly chosen for the effectiveness of stable filter design procedure. It should be noted that to our knowledge no solution has been proposed yet for the choice of appropriate value for IIR filter order and group delay for a given design specification.

In our previous work [14] we have introduced formulas (for quasi-equiripple IIR filter design) that provided estimated filter order and minimum group delay τ_{min}. The formulas can be applied if the desired frequency response is of the form $D(\omega) = |D(\omega)|e^{j\tau\omega}$ and transfer function $H(z)$ has unequal number of poles and zeros (a few poles outside the origin of the complex variable plane and an arbitrary number of zeros). The estimation of minimum group delay is important because for $\tau \geq \tau_{min}$ a stable filter is obtained. However, a large magnitude overshoot appears in frequency response $H(\omega)$ when τ_{min} is imposed and further looking for appropriate value of the group delay is necessary. In this paper we propose new formula for group delay estimate that guarantees filter stability and small magnitude overshoot (usually less than 1 dB). Improved estimate of filter order is also delivered below. As in [14] we limit our discussion to the filters with unequal number of poles and zeros. Filters of this type are a compromise between computational complexity and features like roundoff noise, coefficient sensitivity, phase linearity [15], [16].

The composition of this paper is as follows: section II presents the problem formulation, whereas section III shows derivation of practical formulas for group delay and filter order estimation. The design examples which illustrate practical usage of proposed formulas are presented in Section IV. Section V contains comment on weighting function that is suitable for both magnitude and phase response approximation, and section VI provides the final conclusions from this work.

II. Problem Formulation

Consider the digital IIR filters described by transfer function:

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{n=0}^{N} b_n z^{-n}}{\sum_{m=0}^{M} a_m z^{-m}} = \frac{\sum_{n=0}^{N} b_n z^{N-n}}{z^{N-M} \sum_{m=0}^{M} a_m z^{M-m}} \quad (4)$$

for the case $M < N$. The goal is to determine order N (for fixed M) and group delay τ of desired transfer function $D(z)$ that guarantee a stable filter satisfying given set of filter specifications for quasi-equiripple approximation of $D(\omega)$. We use passband edge ω_p, stopband edge ω_s and stopband attenuation A_s as specifications (for lowpass filters). The following two assumptions are also made: firstly, the weighting function $W_0(\omega)$ is equal to one in the passband as well as in the stopband, and $W_0(\omega) = 0$ in the transition band; secondly, the magnitude overshoot in the transition band should be less than 1 dB. Some comments about other values of the weighting function $W_0(\omega)$ in the passband and stopband are made in the Section V.

III. Derivation of Practical Formulas

The practical formulas for N and τ have been obtained using algorithm [11] for filter design. Numerous experiments have been carried out, with various values of τ, N, ω_p and ω_s. Hundreds of filters have been designed assuming $M = 2$, 4 or 6 in case of lowpass filters, and $M = 4$ or 8 in case of bandpass filters, for even N ranged from 8 to 50. Passband width ω_{pw} and transition band width $\omega_t = |\omega_s - \omega_p|$ were also changed during experiments. We started from $\tau = \tau_{min}$ [14] and successively increased until $\tau = \tau_1$ for which the magnitude overshoot would be less than assumed value of 1 dB. Next, we determined the minimum stopband attenuation A_s of the filter based on frequency magnitude response. As a result two relationships were obtained: $\tau_1 = f_1(N, \omega_t, \omega_{pw})$ and $A_s = f_2(N, \omega_t, \omega_{pw})$ for each of the designed filter. The conclusions from experiment are as follows: in case of lowpass filters with $M = 2$ and bandpass filters with $M = 4$ the relationship $\tau_1 = f_1(N, \omega_t, \omega_{pw})$ is linear in N and the slope of this line depends on transition band width ω_t (see Fig. 1). Filter attenuation A_s is a nonlinear function of N and depends on ω_t as well (Fig. 2). For lowpass filters with $M = 4$ or 6 and bandpass filters with $M = 8$, an additional effect of the passband width ω_{pw} can be observed in both f_1 and f_2 relationships (see Fig. 3 and Fig. 4), which must be taken into account if ω_{pw} is less than 0.2 for lowpass filters or less than 0.3 for bandpass filters. In Fig. 1 and Fig. 2 average values of τ and A_s have been plotted in order to emphasize real changes of the relationships because functions f_1, f_2 in case of lowpass filter with $M = 2$ are independent of the filter passband width ω_{pw}.

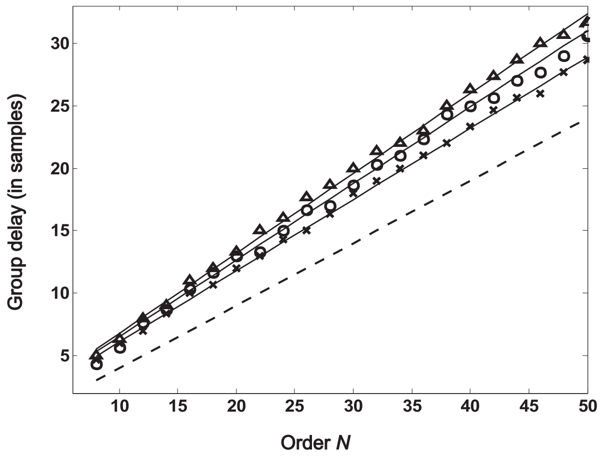

Fig. 1. The average values of τ_1 versus N for lowpass filters with $M = 2$ obtained for: $\omega_t = 0.04\pi$ (△), $\omega_t = 0.07\pi$ (o), $\omega_t = 0.16\pi$ (x), and its approximation (solid lines) by (5), τ_{min} plotted for comparison (dashed line).

The following linear function has been proposed for f_1 approximation:

$$\tau_1 = \alpha(\omega_t, \omega_{pw})N + \beta(\omega_t, \omega_{pw})\,. \quad (5)$$

Two stage procedure was applied for determining the coefficients $\alpha(\omega_t, \omega_{pw})$ and $\beta(\omega_t, \omega_{pw})$. At first, least squares approximation of $\tau_1 = f_1(N, \omega_t, \omega_{pw})$ was used for calculation of α and β for filters with various ω_t and ω_{pw}. In the second stage, the relationships α and β as functions of ω_t and ω_{pw} were modelled by means of second order polynomials $\alpha(\omega_t, \omega_{pw})$, $\beta(\omega_t, \omega_{pw})$ (5). Matlab functions *polyfit* and *lsqnonlin* have been applied for calculation.

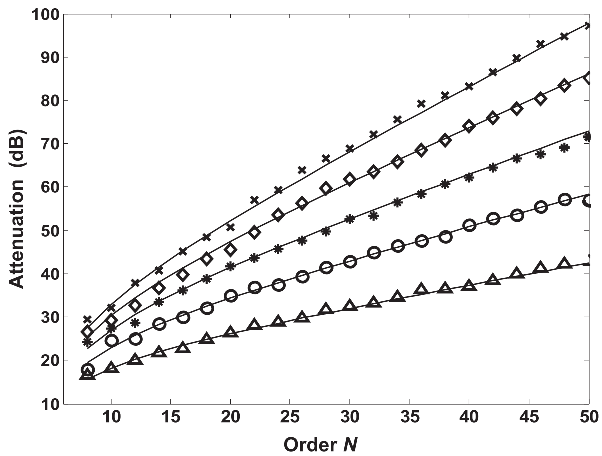

Fig. 2. Approximation of the average values of the stopband attenuation versus N for the lowpass filters with $M = 2$ and with ω_t as parameter: $\omega_t = 0.04\pi$ (△), $\omega_t = 0.07\pi$ (o), $\omega_t = 0.1\pi$ (*), $\omega_t = 0.13\pi$ (◇), $\omega_t = 0.16\pi$ (x).

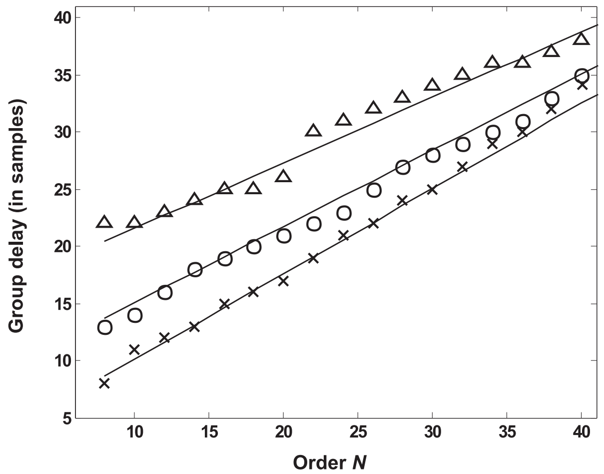

Fig. 3. The values of τ_1 versus N for lowpass filters with $M = 4$ and $\omega_t = 0.04\pi$ obtained for: $\omega_{pw} = 0.05\pi$ (△), $\omega_{pw} = 0.1\pi$ (o), $\omega_{pw} = 0.2\pi$ (x), and its approximation (solid lines) by (5).

TABLE I
THE FORMULAS FOR α AND β CALCULATION FOR LOWPASS FILTERS

M	α	β
$M = 2$	$0.35\omega_t^2 - 0.409\omega_t + 0.685$	0.41
$M = 4$ $\omega_{pw} > 0.2\pi$	$0.480\omega_t^2 - 0.737\omega_t + 0.94$	$-3.87\omega_t^2 + 6.79\omega_t$ -1.28
$M = 4$ $\omega_{pw} \le 0.2\pi$	$0.439\omega_t^2 - 0.546\omega_t + 0.41$ $+1.51\omega_{pw} - 0.089\omega_{pw}\,\omega_t$ $-1.25\omega_{pw}^2$	$7.17\omega_t^2 - 23.4\omega_t +$ $29.8 + 64.5\omega_{pw}^2 -$ $85.0\omega_{pw} + 33.3\omega_{pw}\,\omega_t$
$M = 6$ $\omega_{pw} > 0.2\pi$	$(-1.32\omega_{pw} + 1.91)\omega_t^2$ $+0.78\omega_{pw}\omega_t - 1.34\omega_t$ $+0.022\omega_{pw} + 0.821$	$(-3.73\omega_{pw}^2 + 14.65\omega_{pw}$ $-14.38)\omega_t + 4.41\omega_{pw}$ $-18.59\omega_{pw} + 18.97$
$M = 6$ $\omega_{pw} \le 0.2\pi$	$0.562\omega_t^2 - 2.08\omega_t$ $+0.862 - 0.772\omega_{pw}^2 +$ $0.384\omega_{pw} + 2.5\omega_{pw}\omega_t$	$14.49\omega_t^2 - 6.13\omega_t$ $+39.33 + 67.8\omega_{pw}^2 -$ $88.98\omega_{pw} - 16.7\omega_{pw}\omega_t$

ω in radians per sample

Nonlinear relationship f_2 can be approximated by several functions; for instance the second order polynomial has been proposed in [14] for N ranging from 8 to 50. In this paper we propose another function:

$$A_s = \lambda(\omega_t, \omega_{pw})N + \delta(\omega_t, \omega_{pw}) + \frac{\gamma(\omega_t, \omega_{pw})}{N}. \quad (6)$$

With this function satisfactory estimates of A_s can be obtained even for N greater than 50. Coefficients $\lambda(\omega_t, \omega_{pw})$, $\delta(\omega_t, \omega_{pw})$ and $\gamma(\omega_t, \omega_{pw})$ have been calculated by the same method as $\alpha(\omega_t, \omega_{pw})$ and $\beta(\omega_t, \omega_{pw})$. Table I - IV present coefficients of the approximation functions (5) and (6) derived from experiment: Table I and Table II for lowpass filters, whereas Table III and Table IV for bandpass filters.

The limits imposed on ω_t, ω_{pw} and N result from various criteria. One of the major problems is the number of adders and multipliers needed for filter realization. It can be shown [17] that for quasi-equiripple IIR filters with small M computational saving can be achieved when compared with FIR filters (designed by Parks McClellan method) if $\omega_t < 0.2\pi$, therefore all experiments were performed for $\omega_t < 0.2\pi$. The lower limit of N was chosen according to required minimum filter attenuation. In case of $N < 8$ for lowpass filters, or $N < 10$ for bandpass filters, the stopband attenuation is 20 dB or less, which usually cannot be accepted. The upper limit of $N = 50$ was assumed due to limited simulation time, however formula (6) yields good estimate of A_s even up to $N = 80$, for $M = 2$. The limits for ω_t, ω_{pw} result directly from experiment:

- in case of lowpass filters with $M = 2$:

$$0.01\pi < \omega_t < 0.2\pi, \;\; \omega_{pw} > 0.02\pi, \;\; \omega_t + \omega_{pw} < 0.98\pi,$$

- in case of lowpass filters with $M = 4$ and $\omega_{pw} > 0.2\pi$:

$$0.04\pi \le \omega_t \le 0.13\pi, \;\; \omega_t + \omega_{pw} \le 0.87\pi,$$

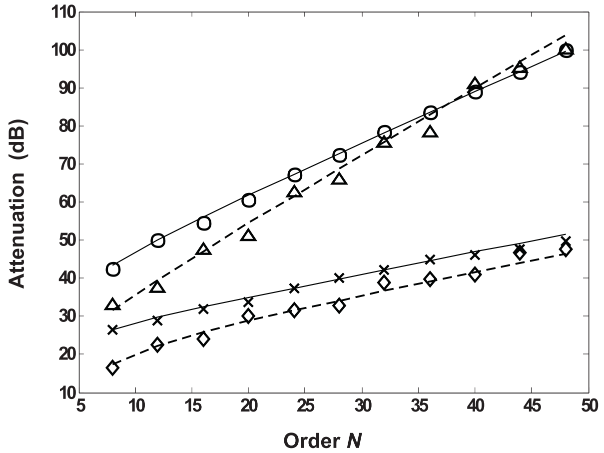

Fig. 4. Approximation of the stopband attenuation versus N for the lowpass filters with $M = 6$ and with: $\omega_{pw} = 0.7\pi$, $\omega_t = 0.13\pi$ (△), $\omega_{pw} = 0.2\pi$, $\omega_t = 0.13\pi$ (o), $\omega_{pw} = 0.2\pi$, $\omega_t = 0.04\pi$ (x), $\omega_{pw} = 0.7\pi$, $\omega_t = 0.04\pi$ (◇).

TABLE II
THE FORMULAS FOR λ, δ AND γ CALCULATION FOR LOWPASS FILTERS

M	λ	δ	γ
$M = 2$	$2.50\omega_t + 0.17$	$-88.0\omega_t^2 + 11$ $+79.6\omega_t$	$486\omega_t^2 - 18.2$ $-407\omega_t$
$M = 4$ $\omega_{pw} > 0.2\pi$	$-5.99\omega_t^2 + 0.03$ $+5.53\omega_t$	$176\omega_t^2 + 20.5$ $-37.5\omega_t$	$-1848\omega_t^2 - 107$ $+677\omega_t$
$M = 4$ $\omega_{pw} \le 0.2\pi$	$-1.94\omega_t^2 - 0.05$ $-1.49\omega_{pw}^2$ $+1.13\omega_{pw}$ $+3.01\omega_{pw}\omega_t$ $+2.39\omega_t$	$3.78\omega_t^2 + 32.6$ $+88.7\omega_{pw}^2$ $-79.0\omega_{pw}$ $-103\omega_{pw}\omega_t$ $+80.6\omega_t$	$389\omega_t^2 - 34.4$ $-256\omega_{pw}^2$ $+143\omega_{pw}$ $+27\omega_{pw}\omega_t$ $+36.3\omega_t$
$M = 6$	$-1.85\omega_t^2 - 0.05$ $+3.03\omega_t$ $-0.19\omega_{pw}^2$ $+0.41\omega_{pw}$ $+0.96\omega_{pw}\omega_t$	$-2.51\omega_t^2 + 32.8$ $+62.7\omega_t$ $+10.2\omega_{pw}^2$ $-28.1\omega_{pw}$ $-27.0\omega_{pw}\omega_t$	$-447\omega_t^2 - 21.0$ $+125\omega_t$ $-3.58\omega_{pw}^2$ $-27.2\omega_{pw}$ $+104\omega_{pw}\omega_t$

ω in radians per sample

TABLE III
THE FORMULAS FOR α AND β CALCULATION FOR BANDPASS FILTERS

M	α	β
$M=4$	$0.512\omega_t^2-0.468\omega_t$ $+0.684$	-0.45
$M=8$ $\omega_{pw}>0.3\pi$	$0.420\omega_t^2-0.615\omega_t$ $+0.883$	$-3.23\omega_t^2+7.43\omega_t$ -2.38
$M=8$ $\omega_{pw}\le 0.3\pi$	$1.71\omega_t^2-0.873\omega_t$ $+0.462-0.116\omega_{pw}\omega_t$ $+0.608\omega_{pw}-0.219\omega_{pw}^2$	$-36.5\omega_t^2-1.47\omega_t+23$ $-37.2\omega_{pw}+15.4\omega_{pw}\omega_t$ $+13.9\omega_{pw}^2$

ω in radians per sample

- in case of lowpass filters with $M = 4$ and $0.05\pi \le \omega_{pw} \le 0.2\pi$:

$$0.04\pi \le \omega_t \le 0.13\pi,$$

- in case of lowpass filters with $M = 6$ and $\omega_{pw} > 0.2\pi$:

$$0.04\pi \le \omega_t \le 0.16\pi,\ \ \omega_t+\omega_{pw} \le 0.87\pi,$$

- in case of lowpass filters with $M = 6$ and $0.1\pi \le \omega_{pw} \le 0.2\pi$:

$$0.04\pi \le \omega_t \le 0.16\pi,$$

- in case of bandpass filters with $M = 4$:

$$0.04\pi < \omega_t < 0.2\pi,\ \omega_{pw} > 0.05\pi,\ 2\omega_t+\omega_{pw} < 0.92\pi,$$

- in case of bandpass filters with $M = 8$ and $\omega_{pw} > 0.3\pi$:

$$0.04\pi \le \omega_t \le 0.13\pi\ \ 2\omega_t+\omega_{pw} \le 0.84\pi,$$

- in case of bandpass filters with $M = 8$ and $0.15\pi \le \omega_{pw} \le 0.3\pi$:

$$0.04p \le \omega_t \le 0.1\pi.$$

The proposed method was introduced for lowpass and bandpass filters. However, formulas (5), (6) can be also applied for the highpass and bandstop filters, with the same passband width ω_{pw}. In order to evaluate the accuracy of the proposed approximation, the group delay error ε_τ and the minimum stopband attenuation error ε_A were calculated. Both errors were defined as the difference between the values obtained by approximation formulas (5) or (6) and the real values. In particular, the acceptance limit for group delay error was $|\varepsilon_\tau| \le 1$ sample, whereas for stopband attenuation error ε_A two levels were considered: $|\varepsilon_A| < 1$ dB and $|\varepsilon_A| > 2$ dB. Table V contains the percentage of filters resulting from design procedure for which the given conditions were satisfied (LP and BP denote lowpass bandpass filters respectively). The results show that the proposed formula (5) successfully approximates the group delay for all types of filters. Quality of approximation of the minimum stopband attenuation depends on filter type; however, in the worst case about 77% of lowpass filters and 73% of bandpass filters satisfy $|\varepsilon_A| \le 2$ dB. In some cases, when filters designed by use of (5) and (6) do not meet given specifications, one can try to increase group delay if the magnitude overshoot is too large, or increase N if the stopband attenuation is too small. However, as these parameters are not independent, modification of any of them may lead to the modification of another one.

TABLE IV
THE FORMULAS FOR λ, δ AND γ CALCULATION FOR BANDPASS FILTERS

M	λ	δ	γ
$M=4$	$3.89\omega_t^2+0.4$ $+0.37\omega_t$	$-255\omega_t^2+0.62$ $+163\omega_t$	$1831\omega_t^2+31.9$ $-1113\omega_t$
$M=8$ $\omega_{pw}>0.3\pi$	$-3.1\omega_t^2+0.06$ $+4.23\omega_t$	$37.5\omega_t^2+14.4$ $+25.0\omega_t$	$-490\omega_t^2-42.0$ $-67.0\omega_t$
$M=8$ $\omega_{pw}\le 0.3\pi$	$-3.07\omega_t^2+0.7$ $+2.42\omega_t$ $+0.92\omega_{pw}^2$ $-1.44\omega_{pw}$ $+1.9\omega_{pw}\omega_t$	$112\omega_t^2-2.98$ $+41.0\omega_t$ $-43.6\omega_{pw}^2$ $+59.9\omega_{pw}$ $-58.6\omega_{pw}\omega_t$	$-1143\omega_t^2+278$ $+175\omega_t$ $+561\omega_{pw}^2$ $-832\omega_{pw}$ $+117\omega_{pw}\omega_t$

ω in radians per sample

TABLE V
APPROXIMATION ERRORS IN STATISTICAL COMPARISON

Filter type	Number of filters	Percentage of filters for which [%]		
		$\lvert\varepsilon_\tau\rvert \le 1^*$	$\lvert\varepsilon_A\rvert \le 1$dB	$\lvert\varepsilon_A\rvert > 2$dB
LP $M=2$	330	98.2	65.5	12.0
LP $M=4$ $\omega_{pw}>0.2\pi$	176	98.9	52.3	14.8
LP $M=4$ $\omega_{pw}\le 0.2\pi$	440	91.8	62.9	6.1
LP $M=6$ $\omega_{pw}>0.2\pi$	163	92.6	43.6	26.4
LP $M=6$ $\omega_{pw}\le 0.2\pi$	175	92.1	43.0	22.4
BP $M=4$	252	95.2	44.8	17.5
BP $M=8$ $\omega_{pw}>0.3\pi$	168	98.8	33.9	26.8
BP $M=8$ $\omega_{pw}\le 0.3\pi$	252	93.3	65.9	5.6

$*$ in samples

IV. DESIGN EXAMPLES

To illustrate the practical usage of the proposed formulas let us consider two examples.

A. Example 1

We want to design the IIR lowpass filter with $M = 6$ and linear phase in passband satisfying the following specifications: passband edge $\omega_p = 0.5\pi$, stopband edge $\omega_s = 0.6\pi$, stopband attenuation $A_s = 34$ dB. The same filter was designed in [8] by means of semidefinite programming. We want to check whether the values N and τ_1 obtained by (5), (6) coincide with those applied in [8]. First, the coefficients λ, δ, γ have to be evaluated by substituting 0.5π for ω_{pw} and 0.1π for ω_t in appropriate formulas from Table II. Next, based on (6) we obtain the following equation

$$34 = 1.38N + 19.89 - \frac{26.2}{N} \tag{7}$$

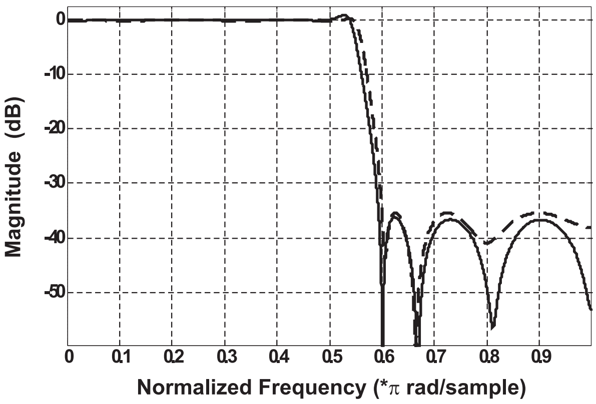

Fig. 5. Magnitude frequency response of lowpass filters designed by algorithm from [11] (solid line) and from [8] (dashed line).

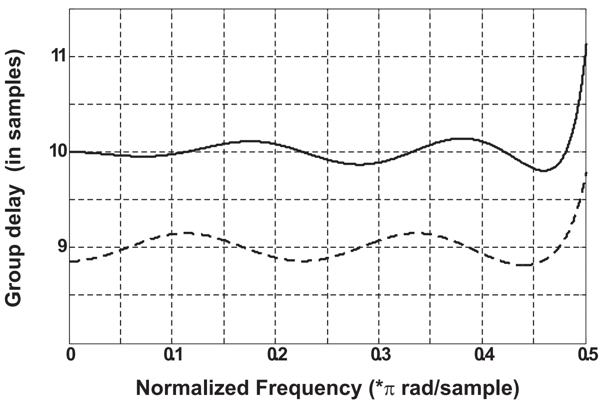

Fig. 6. Group delay response in passband for lowpass filters designed by algorithm from [11] (solid line) and from [8] (dashed line).

which has two solutions. We choose the positive one 11.83, which after rounding gives $N = 12$. Then from (5) we calculate group delay $\tau_1 = 10$. Now we have all data to start the optimization algorithm [11]. Figure 5 and Fig. 6 show magnitude frequency response and group delay response of the designed filter. The magnitude overshoot is 0.91 dB and attenuation A_s in stopband is 36.2 dB. The filter's performance was similar to the filter in [8] (dashed line in Fig. 5) obtained for $N = 12$ and $\tau = 9$. Thus the values N and τ_1 obtained by means of formulas (5), (6) are good estimates.

B. Example 2

Design the IIR bandstop filter with $M = 4$, linear phase in passband and passband edges $\omega_{p1} = 0.4\pi$, $\omega_{p2} = 0.6\pi$, stopband edges $\omega_{s1} = 0.45\pi$, $\omega_{s2} = 0.55\pi$, stopband attenuation 40 dB. Applying formulas (5), (6) we obtain $N = 40.36$ and $\tau_1 = 24.48$. Thus after rounding $N = 40$, $\tau_1 = 24$. Figure 7 shows the magnitude frequency response of the designed bandstop filter. The resulting attenuation in the stopband is 39.87 dB and the magnitude overshoot is 0.8 dB.

V. Comments on Weighting Function

Let us come finally to the question of the weighting function $W_0(\omega) \neq 1$ in the passband or in the stopband. We use constant value of this function thus we denote $W_0(\omega) = W_p$

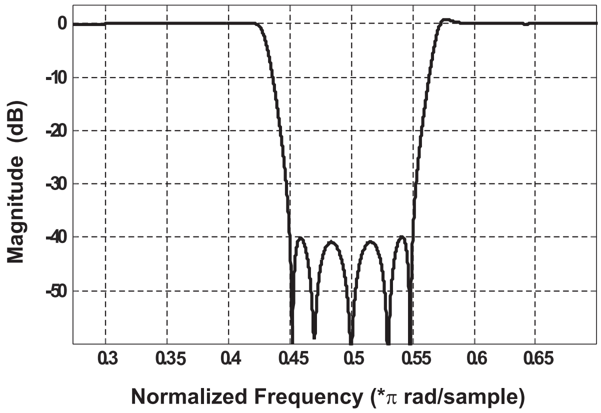

Fig. 7. Magnitude frequency response of the bandstop filter.

in the passband and $W_0(\omega) = W_s$ in the stopband. If $W_p/W_s = K < 1$ and the algorithm [11] is applied, then the stopband ripple is K times less than the passband ripple, but the exact value of these ripples are unknown. Our experience allow to affirm that the formula (5) is still satisfied in case of $K \neq 1$ and the value of N calculated by formula (6) must be shifted up or down depending on K (see Fig. 8). It is clear from Fig. 8 that we can increase stopband attenuation of the filter assuming $W_s > 1$ whereas $W_p = 1$. Unfortunately, at the same time group delay ripple increases as well. We prefer to decrease group delay ripple because our goal is to design linear phase IIR filter. If we assume $W_p > 1$ and $W_s = 1$, it causes the decrease of the stopband attenuation. Hence, in our opinion the assumption $W_s = W_p = 1$ is a good compromise for the filters to be considered.

VI. Conclusion

The useful formulas to calculate the group delay and the order N of the IIR filter were presented in the paper. Good results of estimation are achieved for the lowpass and highpass filters with two, four or six poles (outside the origin of the z-plane) and for the bandpass and bandstop filters with four or

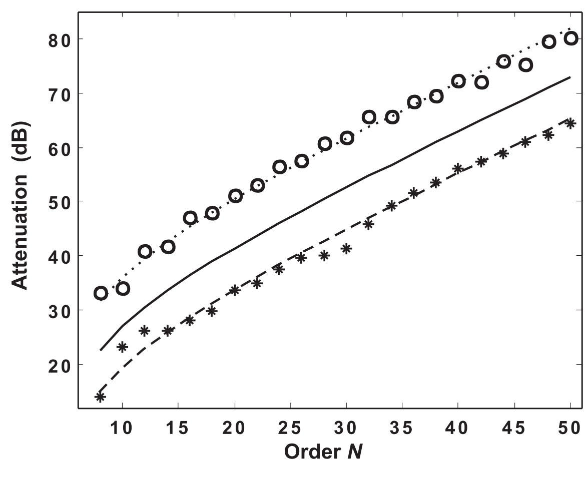

Fig. 8. Relationship $A_s = f_2(N)$ for lowpass filters with $M = 2$, $\omega_t = 0.1\pi$, $\omega_p = 0.4\pi$ and $W_p = 1$, $W_s = 10$ (o), $W_p = 5$, $W_s = 1$ (*). Solid line denotes approximation by (6) for $W_p = W_s = 1$, dotted line=solid line+9 dB, dashed line=solid line-6 dB.

eight poles. We use even number of poles because it is good practice to assign one pole for each transition band [18] (if ω ranges from 0 to 2π). The number of zeros is arbitrary for all types of filters. The region of validity of the derived formulas depending on passband width, transition band width and filter order is also determined. Outside the validity region formulas (5), (6) can be used for obtaining starting values of τ_1 and N, followed by search for the optimal ones. Moreover, both derived formulas are not restricted to the algorithm from [11] but are suitable for all quasi-equiripple design.

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