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On Scientific Realism and Instrumentalism in Manoeuvring Target Modelling and Tracking

Mirosław Sankowski

***Abstract*—The basic problem of tracking manoeuvring moving objects (e.g. aircrafts, ships) lies in unpredictability of object manoeuvres, with respect to the time of occurrence, duration and the type of trajectory. In this paper most representative methods of modelling and state estimation techniques applied to Manoeuvring Target Tracking (MTT) are briefly reviewed. Classification of existing approaches is made in the context of realistic and instrumentalistic paradigms of the philosophy of science. A practical example is also given that shows the impact of selecting models and estimation methods on the performance of the tracking filter for Air Traffic Control (ATC) radar.**

***Keywords*—radar tracking, manoeuvring targets, modelling, state estimation, scientific realism/instrumentalism.**

I. Introduction

DIFFERENT solutions to the problem of tracking manoeuvring targets have been proposed in the vast literature on the subject. They are covered to varying degrees in major books [4], [9], [10], [27] and surveys [26], [51], [53]. Rong Li and Jilkov in their reviews [51]–[53] proposed a classification of Manoeuvring Target Tracking (MTT) methods using two criteria: filter mechanisation and manoeuvre modelling methodology.

Clearly, modelling of aircraft trajectories is a key point in the design of state estimators of any kind, because the quality of the model significantly influences the resulting accuracy of the estimator. On the other hand, it can be shown that state estimation methods, especially those based on the Multiple Model (MM) principle, can be based on models differing strongly in both; the underlying philosophy and mathematical complexity. What is more, essentially different MM estimation methods can rely on the same set of models [36]. Therefore, the classification presented here, which starts from discussing target modelling problems, makes distinctions between the presented filtering methods based exclusively on concepts and mechanisation of the filtering process (e.g. adaptive vs. non-adaptive, single-model vs. multiple-model).

The novelty of this review is related to the presentation framework of modelling and filtering approaches, which is based on the two opposite paradigms of realism and instrumentalism brought up by the philosophy of science.

A. Realistic versus Instrumental Models

The problem of modelling physical phenomena is a subject discussed by philosophy of science [16]. Scientific *realism* is a view that our best scientific theories really refer to unobservable entities, which exist independently of our minds, and that the universe really is as science describes it. It holds that the elements of the universe such as subatomic particles, microbes, stars, electromagnetic force and so forth exist independently of observation or perception. *Instrumentalism* (anti-realism) in the philosophy of science, is any view which denies that we may know that our theories refer to mind-independent unobservable entities, because the eternal truth is not within our grasp. It says that concepts and theories are merely useful instruments whose worth is measured not by whether the concepts and theories are true or false (or correctly depict reality), but how effective they are in explaining and predicting phenomena. Probably the most popular and successful instrumental approach to modelling is based on probability theory [45]. An influence of the two opposite doctrines can be also discerned in various approaches to the problem of modelling aircrafts' trajectories.

Mirosław Sankowski is with Przemysłowy Instytut Telekomunikacji S.A., ul. Hallera 233A, 80-502 Gdańsk, Poland (e-mail: miroslaw.sankowski@ieee.org).

B. Basic Definitions

In order to make the presentation simple our discussion is limited to discrete-time dynamic systems. Various types of target motions can be described by the following linear state-space model (system equation)

$$\boldsymbol{x}[n+1] = \boldsymbol{F}\boldsymbol{x}[n] + \boldsymbol{B}\boldsymbol{u}[n] + \boldsymbol{G}\boldsymbol{w}[n] \tag{1}$$

where $\boldsymbol{x}[n]$ is a state vector, $\boldsymbol{u}[n]$ is a control input, $\boldsymbol{w}[n]$ represents a process noise, $\boldsymbol{F}$ is a state transition matrix, while $\boldsymbol{B}$ and $\boldsymbol{G}$ denote matrices projecting, respectively, the control input and the process noise into the model state-space. Matrix $\boldsymbol{F}$ in (1) usually describes a non-manoeuvring motion, the control $\boldsymbol{u}[n]$ models a manoeuvre, while the noise $\boldsymbol{w}[n]$ accounts for modelling errors.

The control input $\boldsymbol{u}[n]$ is primarily deterministic in nature and typically unknown to the tracking algorithm [52]. Thus a natural way is to model it as an unknown, deterministic process. When the design of the model (the control input and the respective projection matrix $\boldsymbol{B}$) aims at imitating the real geometry of the target movement in space using known models of physical phenomenons, this methodology can be referred to as *realistic*. Good examples of this approach are the methods based on flight dynamics discussed at the end of this section. Alternatively $\boldsymbol{u}[n]$ can be modelled as a random process. Although the use of statistical methods for describing deterministic phenomenons do not intend to literally reflect them, it may lead to a convenient mathematical foundation for effective numerical techniques (algorithms). This approach, which aims at obtaining efficient algorithms rather than accurate models, can be viewed as an example of the *instrumental* approach.

II. Modelling of Manoeuvring Aircrafts' Trajectories

Dynamic models of manoeuvring targets' movements are, in general, non-stationary regarding both their structure and parameters. Although any real target is characterised by its shape and dimensions, due to observability conditions in radar tracking problem it is usually treated as a point object. As a result, target motion models describing the evolution of the target state with respect to time are in fact kinematic models.

An important issue related to movement modelling is the choice of a coordinate system, which constitutes basic framework for mathematical models. Depending on the selected coordinate system, the resulting model can be simple or sophisticated, linear or non-linear, coordinate coupled or uncoupled, and finally, can lead to a well or poorly conditioned state-estimation problem.

Limited discussion has been devoted to modelling non-manoeuvring (uniform) motions of aircrafts. Because the majority of the considered models neglect the Earth's curvature, practical models of uniform motion usually rely on a straight-line constant-velocity principle, commonly referred to as a *Constant Velocity* (CV) models [6], [58]. The CV model is usually formulated in a suitable radar-related Cartesian coordinate system, in which it is uncoupled and linear.

The comprehensive review of modelling techniques used for the MTT applications was given by Rong Li and Jilkov [52]. In this section a short survey of major aircraft's manoeuvre modelling techniques is made which constitutes a necessary background for presentation of the MTT techniques in the subsequent part of this paper.

A. Instrumental Stochastic Models

Stochastic models of manoeuvres can be roughly classified into the following groups [52]. *White noise acceleration models* assume that the target acceleration is an independent process (white noise). It differs from the non-manoeuvring target model (CV) only in noise intensity, which must be sufficiently high in order to account for a manoeuvre. In the third group the control input is modelled as a Markov process, as in the Singer model [58], which assumes that the target acceleration is a zero mean first-order stationary Markov process (coloured noise) characterised by exponential autocorrelation function. The third group utilises *semi-Markov jump process* models for describing the control input, which is equivalent to modelling the acceleration as a stochastic process with a switched mean. The noise itself can be either white [28] or coloured [42] in this case.

Stochastic models of manoeuvres usually do not reflect any geometry of the manoeuvre. As a result, they neglect the fact of coupling among target coordinates, which is very attractive from the computational aspect and makes allowances for separating the tracking problem into two or three simultaneous scalar tracking processes. On the other hand, the statistical model of trajectory is not applicable for making prediction of target positions during manoeuvres. Therefore, a great effort has been devoted to model manoeuvring trajectories using analytical (deterministic) models.

B. 2D (Planar) Models

Basic types of aircrafts' movements are those, which can be approximated by straight lines, including the CV model already introduced. When a target moves along a straight line with a constant *longitudinal* acceleration, the motion can be described by a *constant acceleration* (CA) model.

In the Cartesian coordinates the CA model can be described as a 2-rd order polynomial of time. This results in a linear and uncoupled model, which is often used to model uniform changes of the aircraft's speed. Because the Kalman filter based on the CA model can be characterised by relatively short memory, it is sometimes used for tracking all kinds of manoeuvres [1]. Clearly, the use of the CA model as a foundation for tracking, for instance, circular turns, is an example of the instrumental use of this model.

Aircraft turns can be modelled as circular target motions with the constant turn rate occurring (usually but not necessarily) in a horizontal plane. The resulting trajectory has a constant curvature, while acceleration and velocity vectors are perpendicular. Such a model is referred to as a *coordinated turn* (CT) model, while an effective acceleration modelling all forces acting on the aircraft body during CT is called *normal* or *transversal*. Various versions of this constant-speed, horizontal CT model were proposed for tracking.

Laňka [37] derived his circular manoeuvre model in the Cartesian coordinates directly from the circle equation. The resulting model, describing the evolution of target positions during horizontal turns, was used for a circular-manoeuvre detection and classification using a bank of matched filters. It is not well suited for the use as a design basis for recursive state estimation, though.

Roecker and McGillem [46] proposed a CT model in a polar, manoeuvre-centred coordinate system. Such model seems to be natural because of its linearity. On the other hand, the manoeuvre-centred coordinates used are inconvenient due to difficulties in integrating filters operating in such coordinates with other filter(s) using models expressed in the Cartesian coordinates, which are common for modelling straight-line movements. Therefore, the Cartesian coordinate systems are useful in modelling coordinated turns, even though the resulting models are coordinate-coupled and non-linear.

Early CT models in the Cartesian coordinates shared the assumption of the constant and known angular speed [21]. Although this approach results in convenient linear models, much work has been devoted to more practical variants of the CT model that make allowances for estimating unknown angular speed on-line [48]. These works were devoted to modelling coordinated turns in state space, using purely Cartesian frames, i.e. with the state vector consisting of Cartesian positional and velocity coordinates. It is possible, however, to use another set of "Cartesian" coordinates, which describe the position of the target in the Cartesian coordinates and its velocity in target originating polar coordinates, including speed and course. It was observed [29] that the Extended Kalman Filter (EKF) using such a "mixed Cartesian" CT model offers better performance when tracking coordinated turns, as compared to the one based on the "purely Cartesian" coordinates.

C. 3D Models

Each of the presented 2D horizontal models can be applied to 3D tracking of civilian aircraft in ATC systems, because such targets usually manoeuvre in a horizontal plane and perform only limited vertical manoeuvres. Thus the altitude changes of civilian aircrafts are often modelled independently by a CV model or a random walk model along the vertical direction, decoupled from the horizontal movement model. On the other hand, when the problem of tracking a military aircraft is considered, capable of performing evasive manoeuvres in the 3D space, such decoupled models may be inadequate.

Much effort has been devoted to the problem of modelling aircrafts' movements in the 3D space [52]. A 3D-CT model proposed by Watson and Blair [66] is not confined to any arbitrary plane and offers an improvement in accuracy as compared to the horizontal CT model if the manoeuvre plane is tilted with respect to the horizontal one. Nabaa and Bishop [43] proposed a non-constant speed 3D-CT model and validated it, together with the constant-speed ones and with the Singer model [58], against real radar measurements of a military aircraft. The performed tests confirmed that the fighters' manoeuvres occur mostly in a plane (usual assumption of CT models), however, the speed of the target is non-constant.

D. Models Based on Flight Dynamics

The basic idea behind the movement models based on flight dynamics is to enhance modelling accuracy by using a rigid-body principles, known flight dynamics relationships (including the influence of aerodynamic lift, drag and gravity), as well as by explicitly accounting for aircrafts' controls. Berg [7] derived a model for predicting future target positions for a radar-based antiaircraft gun-fire control application. The model describes planar (though not necessarily horizontal) movements of a manoeuvring aircraft and requires computing the aircraft thrust and lift accelerations, which are assumed constant during the manoeuvre.

The movement models based on flight dynamics relationships offer substantial improvement in modelling accuracy, however, at the price of a significant increase in non-linearity, state dimension, and number of uncertain design parameters.

III. Non-Adaptive State Estimators

Early techniques applied to MTT, including a two-point extrapolator, Wiener filter, finite memory filter, or an $\alpha - \beta$ filter [19], [30], [59], [60] were based on a single-model, fixed-gain approach. Since the end of sixties tracking filters are principally supported by the Kalman filtering theory [19].

Let the kinematic model of the non-manoeuvring (uniform) motion of a target is described by

$$\boldsymbol{x}[n+1] = \boldsymbol{F}\boldsymbol{x}[n] + \boldsymbol{G}\boldsymbol{w}[n] \qquad (2)$$

which is a simplified form of (1). In the following we will assume that the state transition matrix $\boldsymbol{F}$ reflects the straight-line constant-velocity (CV) motion model. Under certain conditions a Kalman filter (KF) based on model (2) gives the optimum estimate of the state $\boldsymbol{x}[n]$. If the target starts to manoeuvre the model (2) becomes no longer valid. As a result, the estimation process performed by such *mismatched* (non-optimal) filter leads to a biased estimation and, when the duration of such operation is long, even to the filter divergence.

The basic method to overcome these limitations of the KF with regard to non-stationarity of the manoeuvring target model is to find the intensity of an *equivalent noise* $\boldsymbol{w}^*[n]$ that quantifies the error of the model (2) in describing all the target motions, in particular, manoeuvres. This equivalent noise $\boldsymbol{w}^*[n]$ is then used in (2) to replace the original process noise $\boldsymbol{w}[n]$. In practice, the equivalent noise can be either white or coloured. Singer [58] proposed to model the object acceleration as a first-order auto-regressive process, which is integrated into the state space model by state augmentation.

In fact all such simple methods ignore the nature of targets' manoeuvres and lump all modelling errors introduced by the manoeuvres with the equivalent process noise term $\boldsymbol{w}^*[n]$. If the estimator's only concern is to maintain the track (i.e. to acquire and correlate subsequent measurements) and the radar data is free of false alarms, this method may work reasonably well. On the other hand, it is clear that satisfactory performance of such filters during manoeuvres is achieved at the price of estimation accuracy during non-manoeuvring parts of the target trajectory. It is thus expected that better results can be obtained by means of adaptive state estimators.

IV. Single-Model Adaptive State Estimators

Various adaptive methods were studied to solve the MTT problem [26], [51]. Such filters can adapt themselves to varying conditions by detecting changes or estimating unknown parameters of state equations (1) or (2), while extracting the available information from the radar data.

A. Hard-Decision Approach

It seems reasonable to use the observation residuals (innovation process) available in the Kalman filter algorithm based on non-manoeuvring motion model, so as to detect the manoeuvre. It is known that the innovation (measurement prediction errors) sequence of the optimal linear filter is a random white Gaussian sequence with zero mean. If the target undergoes a manoeuvre beginning at a certain moment (manoeuvre onset time), the innovation process of the filter is characterised by a nonzero mean value (bias).

The non-adaptive methods presented in the previous section attempted to remove (or diminish) the bias by appropriately increasing the filter bandwidth for the whole period of operation. In the hard-decision adaptive approach a bias is being detected in the innovation process in order to adapt (modify) the estimation process so as to lessen negative effects of the manoeuvre solely during its occurrence.

Various techniques were proposed to solve the problem of manoeuvre detection in tracking filters [54]. McAulay and Denlinger [40] considered a bias detector based on a bank of filters processing the residuals of the Kalman filter, each one matched to a certain type of manoeuvre. Each manoeuvre model is described by its onset time and constant intensity (acceleration), and describes an exponential growth of the bias in the innovation sequence. The filter that matches the given residual sequence produces the highest output. When the maximum output value exceeds a suitably chosen threshold, the known model of the maximum-output filter provides the characteristics of the manoeuvre. Similar approach was also followed in [37] to account for coordinated turns.

Many practical MTT algorithms are based on a *chi-square* significance test [51]. They employ statistics that is truly or approximately χ^2-distributed. Such statistics is usually based either on measurement residuals (innovation) [1] or on the estimated value of the control input signal (*Input Estimation*, IE) [17]. Simple detectors usually use a fixed-length window of observation over which the decision regarding the manoeuvre presence is being made. The *Extended Input Estimation* (EIE) algorithm [14], [44] is a recursive procedure based on the *Generalised Least Squares* (GLS) analysis of the innovation process in multiple sliding windows of different lengths. This approach, similar to the use of the bank of matched filters proposed in [40], makes allowances for the detection of the manoeuvre occurrence and the estimation of its onset time and intensity. The chi-square test is the most popular one because of its simplicity, even though it is not necessarily optimal in any sense. In fact, the validity of chi-square tests relies on the assumption that individual terms are Gaussian and independent, which is not always valid in practise [51].

A manoeuvre detector can also be based on a *Generalised Likelihood Ratio* (GLR) test [67]. In a GLR-based algorithm the estimates of the control input are computed for all possible manoeuvre onset times. Then, the one that maximises a suitable (generalised) likelihood ratio function, is taken as the input estimate and the corresponding onset-time as the onset-time estimate [67]. The GLR ratio detector does not presume any distribution of the random processes of interest and declares the detection of a manoeuvre if the likelihood ratio exceeds a given threshold. While providing an analytical framework for change detection, the GLR method has its major drawbacks placed in heuristically choosing the detection threshold and a heavy computational burden [51].

Descriptions of less popular manoeuvre detection techniques can be found in [26], [27], [51]. It is worth noticing, that the manoeuvre detection problem is similar to the problem of failure detection in stochastic dynamic systems, and the methods developed in both areas can be easily cross-applied [32].

After a manoeuvre detection procedure completes, another problem arises: *How to modify the state estimation algorithm to account for the manoeuvre?* A simple method is to drop the current (biased) estimates immediately after the manoeuvre detection and to reinitialise the state estimator using the most recent radar measurements [40]. After such procedure, an effective memory of the Kalman filter becomes short for a certain period of time, and if the manoeuvre ends before the filter gain becomes close to its steady state value (long effective memory), the estimation process can be smoothly continued. If not, the manoeuvre will be detected again and the above procedure repeats.

In [17], after the detection of a manoeuvre based on the estimated value of control input, the estimate and an approximation of its uncertainty (i.e. the covariance of the estimation error) are used to correct the state estimate and the respective covariance matrix of the Kalman filter based on the CV model. Then, the estimation process is performed based on the same filter, as if the manoeuvre has definitely ended. The major drawback of this (as well as the previous) approach is that during manoeuvres of long duration the estimation process will be characterised by a repeatedly increasing estimation error (bias due to manoeuvre) which is successively reduced by subsequent manoeuvre corrections.

A more elegant approach was proposed in [37]. Using the innovation process sequence taken from the Kalman filter based on CV model, manoeuvre classification is based on a bank of filters, matched to coordinated turns of different durations and intensities. If the manoeuvre is detected and classified, its parameters are used to correct current estimates produced by the filter and to compensate the model mismatch during the manoeuvre using the control input $\boldsymbol{u}[n]$.

Adaptation of the Kalman filter to a detected manoeuvre can also be done using the *equivalent noise* principle, described in section III. In this case, after detecting the manoeuvre, an equivalent noise $\boldsymbol{w}^*[n]$ of a suitably chosen intensity can be introduced instead of the original system noise $\boldsymbol{w}[n]$. The use of the equivalent noise method jointly with the manoeuvre detection approach avoids the major disadvantage of non-adaptive tracking filters, i.e. a poor estimator accuracy during non-manoeuvring motion parts.

Another group of tracking filters based on detection of manoeuvres use the mechanism of switching among state estimators based on different models of target movements. A closer view on these algorithms is given in section V.

B. Soft-Decision Approach

The adjustment of a KF based on a non-manoeuvring model to a sudden manoeuvre can be done by adapting the filter bandwidth. Such an adaptive filter may use either a *noise identification* technique, which explicitly identifies the process noise statistics used in the filter, or an *adaptive gain* approach, which accounts for the effect of the uncertainty in the noise characterisation on state estimation indirectly without explicit identification of the noise statistics [51].

Four approaches to the identification of unknown covariances can be recognised [41]: *Bayesian approach*, where the Bayes' rule is used to update the a-priori distribution of noise statistics; *maximum likelihood estimation*, where the noise statistics are estimated by maximising their likelihood functions; *correlation methods*, where the noise statistics are related to and determined by an estimated autocorrelation function of the measurement residual sequence; and *covariance matching*, where the noise statistics are estimated based on matching between theoretical and estimated covariances [51].

In the context of MTT a *noise-level adjustment* approach is often used, which relies on a process noise compensation [19]. A representative application of this approach is given in [15]. Measurement residuals (innovations) in each coordinate are normalised, and then filtered in a single-pole filter. The magnitude of the output of this filter, when exceeds a threshold, is used to vary the process noise covariance in the KF. In [23] a *scale factor* is used, calculated from measurement residuals, which represents the current magnitude of process noise covariance. The scale factor level increases during manoeuvres to provide improved tracking.

Relatively few techniques were developed for the gain adaptation approach, which is based on the analysis of measurement residuals (e.g. biasness or orthogonality) and makes allowances for adapting the filter gain according to a certain rule [51].

Note that all the methods described above implement various adaptive versions of the equivalent noise approach described earlier and offer similar advantages as the manoeuvre-detection-based (or hard-decision) methods. Note, however, that it is not obtained by carefully modelling target trajectory, but by suitable adjustment of the filter bandwidth.

V. Multiple-Model State Estimators

From amongst many techniques of adaptive tracking, *Multiple-Model* (MM) methods became quite popular during the last 20 years. In the MM approach it is assumed that the system obeys one of a finite number of modes. Such systems are called *hybrid* because they have both *continuous* (noise) uncertainties and *discrete* (model or mode) uncertainties [5]. *Hybrid estimation* is the estimation of a quantity (a parameter or process) which has continuous and discrete components [53]. A continuous component will be referred to as a *base state* or *state*, while a discrete component will be called a *modal state*. An MM tracking filter is based on a set of kinematic models (modes), which describe certain types of trajectories and constitute a basis for the applied set of state estimators, called partial or mode-matched filters.

For a suitable description of the most important principles of the MM estimation, a simple classification of the existing approaches can be made. Principal rules of a given MM estimation method can be easily recognised by answering two basic questions: *(1) How do partial estimators work? (2) How is the final estimate computed?* The first question leads to a distinction between *competitive* and *cooperative* estimation schemes and to the issue of exchanging information between the component estimators. The other question refers to the problem of combining several estimates obtained from the mode-matched filters into one estimate. This problem is usually solved by selecting the estimate of the most likely filter (*an exclusive approach*) or by appropriately combining (mixing) all component estimates (*a non-exclusive approach*). In view of this classification, the following four basic approaches to the MM estimation problems can be recognised: *cooperative, exclusive* [1], [44], [46], *cooperative, non-exclusive* [12] *competitive, exclusive* [22], *competitive, non-exclusive* [38], [62].

Note that the above classification, although general, is not very useful for a systematic presentation of the practical approaches existing in the literature on the subject. This is due mainly to the fact that the four distinctive approaches are not equally attractive for the considered problem of state estimation in non-stationary systems. Actually, the competitive (or autonomous) MM approach, originally proposed by Magill [38] for uncertain but stationary systems, has been followed rather rarely with regard to the MTT problem. Therefore, the following presentation of practical MM state estimators proposed for tracking manoeuvring aircrafts, will be given within the class of cooperative MM estimators. The MM filters for the MTT problem can be practically based on either the two following basic techniques of managing the architecture of multiple mode-matched filters: operating multiple filters in parallel or switching to the most likely one.

A. Switched/Sequential MM (SMM) filtering

In this approach, several (usually non-manoeuvre and manoeuvre) movement modes are considered, constituting a set of possible modes. Tracking is performed by a single filter based on a selected model at one time. The decision, which model should be used at a given time instant, is made simultaneously with the current state estimation process using information derived from incoming measurements, estimation results (e.g. the current filter innovation) or prior information. Based on this decision the estimation process can be continued using the current filter or another filter based on a more likely model selected from the considered set of possible models.

The SMM approach has three aspects: modelling, decisions, and filtering [51]. Generally, all the models of target's motion discussed in section II can be potentially incorporated into an SMM filtering scheme. Moreover, different filters may be used for different models so as to take advantage of the properties of each model. Decision procedures are usually based on manoeuvre detection techniques, typically those referenced in section IV. Therefore the SMM filtering approach can be also referred to as a *hard-decision* MM technique.

A popular example of the SMM approach is the *variable dimension* filter [1]. It uses two models: CV for non-manoeuvring and CA for manoeuvring portions (modes) of a target trajectory. Because in the applied Cartesian coordinate system both the CV and CA models are linear polynomial models of different order, the two Kalman filters based on these models are characterised by state vectors of different dimensions. Hence, switching between those filters can be interpreted as the change of the order (dimension) of the current filter. The transition from the CV to the CA model is done on the basis of a *chi-square* manoeuvre detector. The estimation process based on the CA model is initiated with the use of most recent measurements, and terminated when the values of the acceleration components of the estimate become sufficiently small. Note that, in fact, the CA model describes a uniform change of the target speed moving along a straight line. In order to cope with circular manoeuvres the bandwidth of the CA filter must be increased to account for prediction errors caused by the CA model mismatch during the turn.

Based on the same principle, Roecker and McGillem [46] proposed a two-model SMM filter based on the CV model for uniform movements and the CT model in polar manoeuvre-centred coordinates for circular turns. It uses the same *chi-square* manoeuvre detector as proposed in [1] and an additional log-likelihood ratio test in order to determine if the circular manoeuvre has occurred. If both tests are passed, the radius and the coordinates of the centre of the manoeuvre circle are estimated from recent measurements using non-linear least squares, and then a new Kalman filter in polar coordinates is initiated. Major drawbacks of such an approach are related to several additional non-linearities introduced into the mechanism of switching between the Cartesian and polar filters, as well as into the measurement equation associated with the manoeuvre-centred CT model. Moreover, it was observed that the filter based on this model becomes unstable when used during non-circular movements. Obviously, such scenario may happen in a real tracking system, given the "random nature" of radar measurements.

It is worth noticing that all algorithms based on detection of manoeuvres (and filters switching) are characterised by common weaknesses. Namely, it was observed that such algorithms usually produce significant estimation error during transitions from the non-manoeuvring to the manoeuvring mode of operation [3], which is due to the fact that the detection threshold preserves the unchanged estimation process scheme until an assumed confidence regarding the manoeuvre presence is achieved. If the threshold is set high, a long delay of detection causes a large estimation error before it can be corrected. A low threshold causes false manoeuvre alarms which can degrade the resulting accuracy, as well. This problem is highly dangerous in the case of intensively manoeuvring targets (e.g. military aircrafts). The choice of a proper setting for the detection threshold is a difficult task. The threshold optimised with respect to typical target manoeuvres may cause the detector to be insensitive to manoeuvres of long duration and low intensity. Moreover, manoeuvre detectors are sensitive to system modelling errors, e.g. to unexpected changes in radar measurement accuracy. In case of such changes (e.g. due to varying radar environment characteristics) the number of false manoeuvre alarms reported by the detector can suddenly increase, and degrade the estimation quality [35].

In order to enhance the performance of the *variable dimension* filter its original algorithm [1] can be integrated with the EIE technique, as proposed in [44]. Application of the EIE algorithm to the detection of manoeuvres in an SMM filter improves the reaction time and reliability of the manoeuvre detection process with regard to manoeuvres of various durations and intensities. Another useful methodology introduced in [20] aims at decreasing the level of false manoeuvre alarms. This method, called a *two-stage decision* logic, consists of two decision logics used in a series. The first decision logic is used to detect any abnormality in the current filter (measurement residuals) to determine a possible manoeuvre onset or termination by comparing a certain measure with the corresponding threshold. If the abnormality is declared, then a "new" (manoeuvre matched) filter is initiated, which runs in parallel with the "current" filter. Then, the second decision logic based on a maximum likelihood test indicates which of the two filters is the most likely one. It can thus be concluded that an improvement of the performance of the *hard-decision* methods can be reached by taking into account multiple hypotheses in parallel and applying certain "softer" decision mechanisms.

B. *Parallel/simultaneous MM (PMM) filtering*

The category of PMM includes those MM state estimation algorithms that operate all the considered mode-matched filters in parallel. Solid surveys of such techniques are available [5], [53]. PMM techniques are usually formulated within the Bayesian framework. Starting with *a-priori* probabilities of each model being correct (i.e. that the considered system is in a particular mode), the corresponding *a-posteriori* probabilities are obtained during the filtering step.

The Bayesian approach to the PMM estimation was introduced by Magill [38] for an unknown stationary dynamic system. The system is assumed to obey one mode of an M-element set during whole considered period of time. The goal is to calculate an optimum estimate of state of such a system without having any *a-priori* knowledge about the "true" mode. If the models used are linear and Gaussian, the optimal filter consists of a bank of M Kalman filters working independently in parallel on their own estimates. Their likelihood functions are used to update model probabilities. The latter model probabilities are used to combine model-conditioned estimates and respective covariances into an overall state estimate [5]. Due to the fact that the partial filters in this approach work simultaneously and independently, such a structure can be referred to as the *Autonomous/competitive* MM (AMM) estimator.

If the assumed set of models includes a correct one and there are no jumps among the models, then the probability of the true mode converges to unity and the AMM filter is able to yield optimal estimates of the system parameters. If the AMM estimator is used in the case of switching modes, certain ad hoc modifications have to be done to prevent the estimation errors of mismatched filters' from growing to unacceptable levels, for instance, by periodic re-initiation of the filters [22].

In order to cope with non-stationary systems it can be assumed that the considered system undergoes switching in time within the assumed set of M modes. Due to possible jumps from one mode to another, a set of possible *mode histories* (sequences of modes) has to be taken into account. Moreover, the number of mode histories grows exponentially with the discrete-time instant n and is equal to M^n. It is usually assumed that the mode-switching is governed by a Markov process (Markov chain) with known time-invariant mode transition probabilities. Based on these basic assumptions and on the total probability theorem, the conditional probability density function of the base/system state is a Gaussian mixture (sum) with the exponentially increasing number of terms [5]. Since each possible mode sequence has to be assigned a corresponding filter, it is clear that the exponentially growing number of partial filters is needed to estimate the base state, which makes the optimum approach impractical. In order to avoid such an increasing system complexity, suboptimal techniques are used in practise.

A *Generalised Pseudo-Bayesian* (GPB) approaches combine the histories of modes that differ in modes older than an assumed insight backward into mode histories. This is usually denoted as GPBi, where i denotes the number of the considered backward steps. The most common approaches include: the GPB1 (first-order GPB) [28], [42], where only the possible modes in the last sampling period are considered, and GPB2 (second-order GPB) [18], which considers all possible modes in the last two sampling periods. These algorithms require, respectively, M and M^2 mode-matched filters that operate in parallel.

Another approximation of the optimal PMM estimation was introduced by Blom an Bar-Shalom [12] that is called an *Interacting Multiple-Model* (IMM) filter. In such an IMM estimator the state estimate is computed for each possible current model using M filters, and each filter uses a different combination of the previous model-conditioned estimates called a mixed initial condition [5], [49]. The input to any of the M partial filters at the beginning of a new filtering cycle is obtained by the interaction among M filters, which consists of "mixing" of the partial estimates from the previous cycle.

It was observed that the IMM estimator is characterised by accuracy similar to GPB2 techniques and with numerical complexity comparable to GPB1 algorithms. The IMM approach has been successfully applied to solve various technical problems [39], [61] such as: state estimation, system identification, MTT, filtering and smoothing, fault detection and isolation, multi-sensor data fusion, robust speech recognition and multirate processing. When applied to the MTT problem, IMM-based tracking filters were shown to outperform virtually all techniques known before, including those based on IE, EIE and on the SMM principle [3].

A further improvement of the IMM estimation accuracy can be obtained by using various higher-order IMM (IMMi) techniques [11], [61], although at the expense of the increased computational burden. In [61] for instance, state estimates of a linear hybrid system are obtained by using mode switching modelled by an i-th order Markov chain on the basis of all possible mode hypotheses over the i most recent sampling periods and requiring M^i filters running simultaneously.

The PMM techniques described above, using the same set of models at all times, can be referred to as a *fixed structure MM* or fixed model-set approach. A major issue in the design of such estimators is the method of selecting a right (or sufficient) set of models [47], [50]. To have reliable results the set of models must "cover" in some sense the possible system modes and at least one of the models must be "close" enough to the system mode in effect at any time. The fixed structure MM algorithms usually perform well for problems that can be handled with a small set of models, however, in many practical situations this requirement is not satisfied. Moreover, as it was shown theoretically in [50], the use of too many models is as bad as the use of too few models. The major reason of the unsatisfactory behaviour of the MM algorithms based on a large model set is that many models in this set are so different (mismatched) from the true system mode (at a particular time) that excessive "competition" among the mismatched models degrades the performance of the estimator and unnecessarily increases its computational burden.

It is easy to point out certain practical tracking problems for which the number of modes that must be matched by the model set within a certain time periods, can be smaller than the set of all possible modes. Representative applications include, for instance, the problem of tracking long range ballistic missiles, which are able to manoeuvre only during boost and final (re-entry) phases of their flight, and the problem of tracking ground moving targets aided by topography information. Such problems can be approached with the recently introduced technique called *Variable-Structure MM* (VSMM) estimation [2], [50].

The VSMM estimator has a two-level structure: multiple model-set sequences at a higher level and multiple model sequences at a lower level. It uses model-set adaptation techniques. which may utilise *a-priori* knowledge (e.g. most likely mode sequences) originating from a general targets characterisation, external aids (e.g. road maps [33]), as well as *a-posteriori* information about the system modes derived from measurements.

VI. Example – Air Traffic Control

Traditionally, tracking algorithms applied within *Air Traffic Control* (ATC) systems were based on fixed-gain or adaptive KFs [4], [9], [26], [27]. Growing air traffic intensity and the related problem of safety of air navigation result in increasing interest in reliability of air surveillance systems. This is closely related to the performance of the ATC radars and associated RDP systems with respect to the estimation accuracy of aircrafts' motion parameters.

In 1990's many efforts were aimed at designing novel radar tracking algorithms for future ATC systems. Various solutions were proposed to take advantage of: new discrete-time models of aircraft turns [48], [64], [65], multi-sensor data fusion [13], [64], [65], [68] and modern MM estimation techniques [13], [48], [64], [65], [68].

A. Recommendations for ATC Radar Data Processing

Certain standards and recommendations for European ATC systems are formulated by the *European Organisation for the Safety of Air Navigation EUROCONTROL*. The EUROCONTROL Standard Document [24] which was issued in 1997, describes requirements for radar surveillance in the provision of ATC systems in *En-Route Airspace* (ERA) and *Major Terminal Areas* (MTA). Particularly, this document defines the technical requirements for *Primary* (PSR) and *Secondary Surveillance Radars* (SSR), as well as the accuracy requirements for related RDP systems in various configurations, including: single PSR (MTA), combined PSR and SSR (MTA), and single or double SSR (ERA). We focus here on the first problem of radar tracking in the MTA using a single PSR.

Based on the presumed characteristic of civil aircrafts [24], it is assumed that their trajectories may consist of travel path motions and some manoeuvres that include:

- a Uniform Motion (UM) at a constant velocity,
- a Uniform Speed Change (USC) with a constant longitudinal acceleration,
- a Standard Turn (ST) with a constant normal acceleration.

Expected accuracies of estimated trajectory parameters are defined in [24] for certain testing scenarios in terms of a *Root Mean Squared* (RMS) error, with respect to: along-trajectory (ALE), across-trajectory (ACE), ground speed (GSE) and course (COE) errors. Let x, y, v and ψ denote, respectively, two positional coordinates, ground speed and course of the target, while $\hat{x}$, $\hat{y}$, $\hat{v}$ and $\hat{\psi}$ their respective estimates. These four errors are illustrated in Fig. 1.

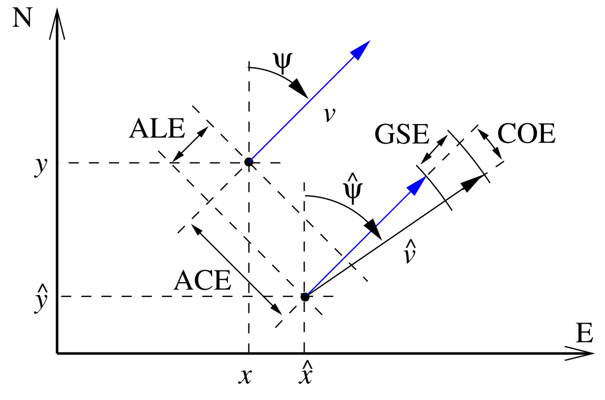

Fig. 1. Geometric illustration of errors: ALE, ACE, GSE and COE.

The required accuracies are described by maximum RMS error defined for steady-states of the three types of motions (UM, USC, ST). During the transitions between different types of motions, the system performance is described by the peak RMS value of the error and the time required for the error to decrease to a reference value that is the fraction of the peak RMS error defined for this type of motion. The values of the maximum and peak RMS errors can be found in [24], [56].

EUROCONTROL recommendation increased the difficulty of designing tracking filters for ATC systems. Traditional requirement of achieving accurate estimates during UMs while maintaining tracks during manoeuvres has changed. Now accuracy requirements are defined for all phases of track life, including manoeuvres, which cannot be met neither by single-model nor by MM filters based on instrumentally increasing filter bandwidth during manoeuvres. Instead, MM methods based on analytical/physical models, describing realistically trajectory shape, must be used.

B. Multiple-Model Approach with Analytical Models

It was often supposed that tracking civilian aircrafts can be successfully performed using a two model IMM estimator based on CV and CT models [31], in which a properly tuned CV model covers UMs and USC manoeuvres, or based on CV/CA models [68], in which the wide band CA model covers USC and ST manoeuvres. Uruski and Sankowski [63] examined three tracking filters based on the IMM technique and: CV and CA models [68], CV and CT models [31] and a new one based on the CV, CA and CT models. The simulation tests showed that it was impossible to tune parameters of either of the two-model IMM estimators to fulfil the EUROCONTROL requirements. Only the three-model IMM filter was close to meet all these requirements.

Similar conclusions were reported by Besada and Garcia [8] who compared two-model IMM filters against the performance requirements for the ARTAS system developed by EUROCONTROL. Then they approached the problem of carefully tuning two- and three-model IMM estimators using evolutionary strategies – a global optimisation technique.

The promising results of applying analytical models of manoeuvres [63] resulted in development of a family of filters based on MM estimation using soft- and hard-decisions [34], a set of trajectory models (CV, CA, CT) and various coordinate systems [36], [56].

Selected results taken from [36], [56] are shown in Fig. 2 and 3 that describe the accuracy of aircraft course estimation for reference trajectories using a three-model IMM filter.

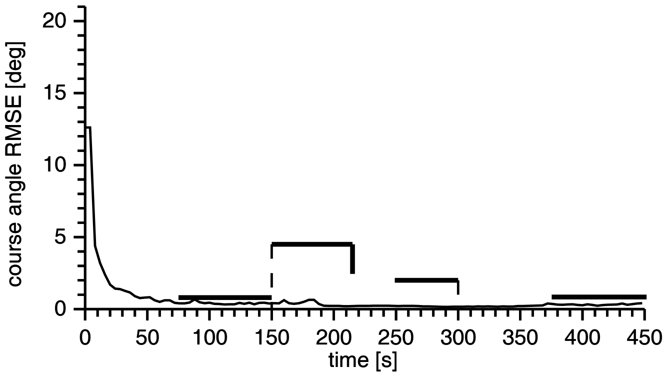

Fig. 2. RMS COE errors for the USC manoeuvre, flat Earth model.

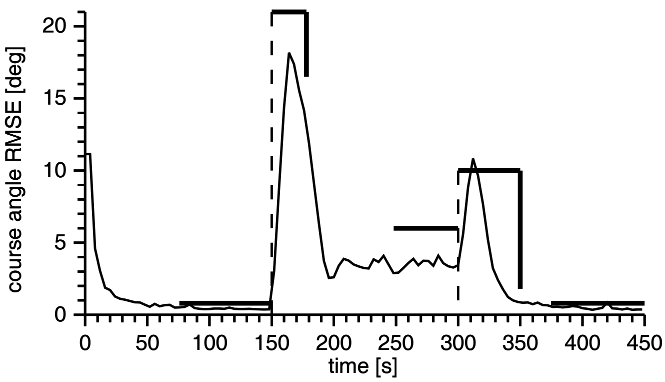

Fig. 3. RMS COE errors for the UT manoeuvre, flat Earth model.

Each of two testing trajectories consist of three parts of equal durations of 150 [s]: (1) UM, (2) USC with longitudinal accel. $1\ [\mathrm{m/s^2}]$ or UT with normal accel. $4\ [\mathrm{m/s^2}]$, (3) uniform motion. Figs. 2 and 3 show the RMS COE errors for a given manoeuvre (USC or CT) based on 500 statistically independent realisations of measurement errors. In figures the thick horizontal lines describe the limiting maximum RMS and peak RMS accuracies, while thick vertical lines indicate the time interval required for decreasing the errors from the peak RMS to the reference RMS value. One can observe that accuracy requirements are met.

C. Modelling the Earth Shape Influence

The above experiment was performed using radar-related Cartesian coordinates for both: simulating the reference trajectory and for estimating the state. It seems obvious, however, that aircrafts navigate by determining their own attitude parameters relatively to the Earth clod, and not with the reference to the radar. In practice, the World Geodetic System 1984 (WGS-84) is widely accepted as a common reference framework for space/airspace navigation and surveillance systems [25].

In Figs. 4 and 5 performance estimates are presented, which describe course RMS errors of the same IMM filter evaluated against trajectory simulated in geodetic coordinates [56].

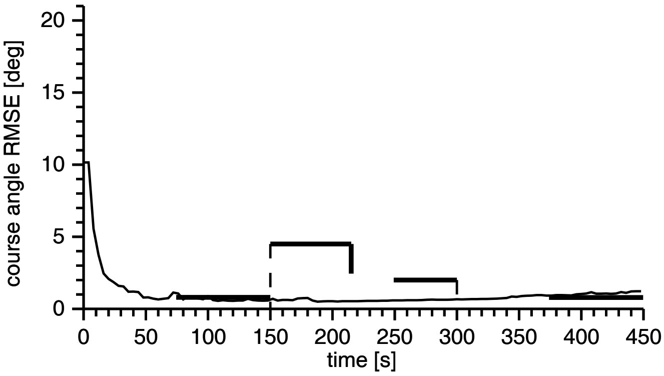

Fig. 4. RMS COE errors for the USC manoeuvre, geodetic Earth model.

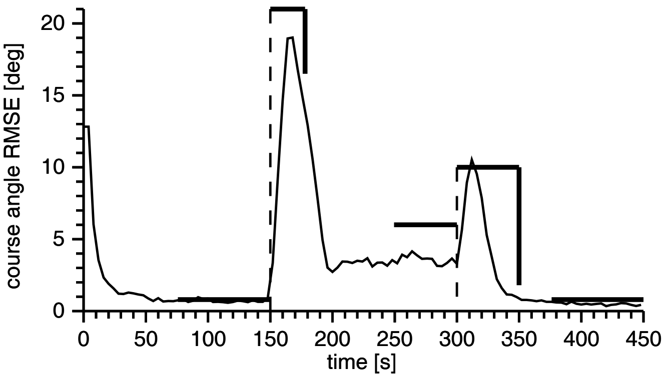

Fig. 5. RMS COE errors for the UT manoeuvre, geodetic Earth model.

The reference aircraft movement model is based on the WGS-84 framework and employs loxodromic UM model, models of UT and USC manoeuvres [56], [57].

One can observe in the figures that now the course estimates suffer from movement models mismatch between the trajectory simulator (geodetic model) and the state estimator (flat Earth Cartesian model). The result of this mismatch is a bias depending on the position of the target with respect to the radar [56]. For some radar coverage regions the value of the bias may exceed the maximum RMS error requirement, which can be observed in the last part of the RMS COE curve in Fig. 4. This bias can [55] and must be corrected in order to reduce the values of the RMS COE error to an acceptable level.

VII. CONCLUSIONS

Concluding this review two observations can be made. First, that the improvement in the state estimation quality of the PMM filters can be obtained by exploiting additional external information. Although the VSMM estimator constitutes a straightforward way of doing this, such knowledge can be also introduced into the fixed model-set MM estimation schemes. It can be done by applying an additional, high-level adaptation mechanism based on tuning the parameters of the Markov chain that describe *a-priori* knowledge on the likelihoods of certain jumps between the modes.

The second observation is related to the evolution of the SMM and PMM approaches. Though the PMM methods generally follow the *soft decision* scheme of adaptive estimation, the VSMM estimator introduces the *hard decision* mechanism of the model-set adaptation. On the other hand, an improvement in the performance of the SMM methods can be achieved by considering competing mode hypotheses in parallel (e.g. two mode-matched filters running simultaneously for some time) and making the model-switching decisions "softer". It can thus be concluded that the two distinctive *hard* (SMM) and *soft* (PMM) approaches somehow converge to a pragmatic method which tries to take advantage of both. Namely, the VSMM filter combines the precise mathematical formulation and the inherent robustness of the PMM techniques, as well as the rationale that real moving objects do not follow trajectories justified by mixing all their possible behaviours (modes), but result rather from a more or less known deterministic sequence of these modes, which is the foundation of all SMM methods.

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