Predictive Maintenance Sensors Placement by Combinatorial Optimization

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Abstract—The strategy of predictive maintenance monitoring is important for successful system damage detection. Maintenance monitoring utilizes dynamic response information to identify the possibility of damage. The basic factors of faults detection analysis are related to properties of the structure under inspection, collect the signals and appropriate signals processing. In vibration control, structures response sensing is limited by the number of sensors or the number of input channels of the data acquisition system. An essential problem in predictive maintenance monitoring is the optimal sensor placement. The paper addresses that problem by using mixed integer linear programming tasks solving. The proposed optimal sensors location approach is based on the difference between sensor information if sensor is present and information calculated by linear interpolation if sensor is not present. The tasks results define the optimal sensors locations for a given number of sensors. The results of chosen sensors locations give as close as possible repeating the curve of structure dynamic response function. The proposed approach is implemented in an algorithm for predictive maintenance and the numerical results indicate that together with intelligent signal processing it could be suitable for practical application.

Keywords—predictive maintenance, optimal sensors placement, combinatorial optimization.

I. INTRODUCTION

R EGULAR condition monitoring and vibration analysis of machinery is a key to predictive maintenance. Carefully planned conditioning monitoring systems prevent machinery failure. Structural health monitoring is an emerging technology, dealing with the development and implementation of techniques and systems where monitoring, inspection and damage detection become an integral part of structures and thus a matter of automation. It does further even merge with a variety of techniques being related to diagnostics and prognostics as such. Structural health monitoring plays a significant role in the fields of automotive, aerospace, mechanical and civil engineering in the last few decades. The strategy of predictive maintenance monitoring is important for successful system damage detection. Maintenance monitoring utilizes dynamic response information to identify the possibility of damage. The basic factors of faults detection analysis are related to properties of the structure under inspection, choice of excitation input signal, and appropriate signal processing. In vibration control, structures response sensing is limited

by the number of sensors or the number of input channels of the data acquisition system. That is why determining the optimal numbers and locations of sensors is a critical issue encountered in the construction and implementation of an effective structural health monitoring system.

Historically, two common maintenance strategies are used in managing equipment: preventive and reactive. The strategy of preventive maintenance estimates the life of the machine based on failure statistics from the same or similar components. Preventive maintenance is based on the availability of statistical data on worst-case failures relevant to usage and time in service. Using statistical safe-life methods to critical systems in most cases leads to conservative estimates about the probability of failure [1].

Numerous investigations are directed towards the study of fault identification. Implementation of condition monitoring system and fault detection system techniques entail initial investment but these costs are being offset by the benefits which are being reaped in the form of continuous production, minimum downtime and more time available for an early planning to replace the defected parts [2]. Flynn and Todd [3] demonstrated that the placement of actuators and sensors is specific to the performance constraints. As such, it is important to accurately define these application-specific constraints before undertaking instrumentation design. A comprehensive survey of available literature on faulted structures was carried out by Doebling et all. [4] to summarize the current state of the detection technology. Guratzsch and Mahadevan [5] defined a methodology for integrating the advances in various individual disciplines for optimum sensor layout design of structural health monitoring systems under uncertainty. The methodology aims at maximizing the probability of detecting damage with respect to the location of structural health monitoring sensors. A problem of locating sensors on a spatial lattice structure with the aim of maximizing the data information so that structural dynamic behavior can be fully characterized is presented [6]. Based on the criterion of optimal sensor placement for modal test, an improved genetic algorithm is introduced to find the optimal placement of sensors. The problem of determining the optimum number of sensors for a particular application, together with their best possible locations, received considerable attention recently. Various methods share a common basis in the recent work on sensor placement in the field of structural dynamics and especially concerned with fault detection [6]. An efficient method based on the uniform design method for optimal sensor placements is developed [7]. It is found that the uniform design method can dramatically reduce the computational

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efforts for optimization. A number of different optimization techniques are developed over the last decades including heuristic approaches, classical and combinatorial optimization [8, 9]. Many early optimization methods, so called ad hoc methods, are based on rough and ready ideas without much use of theoretical background. Stepwise techniques can add or remove one or more sensor at the time in order to find the best combination [10].

For the optimal sensor placement problem it is important not only to find the best positions of sensors for a specific task but also to estimate the required number of sensors for the best sensor performance. Optimal positions deciding and optimal number of sensors defining are two separate problems. The knowledge and experience of engineers are combined with signal processing for the proper solving of optimal sensors locations problem. Designers and end-users of structures know where are the critical machinery areas which need to be analyzed, controlled or monitored. Then an intelligent signal processing could help for the best sensor locations. The problem of optimal number of sensors relies very much on advanced signal processing techniques [11]. From the signal processing point of view, optimal sensor location is optimization and/or selection problem.

In this paper a combinatorial approach based on 0-1 programming for optimal sensors locations is proposed. The sensors should be distributed in such way that all of the required information could be properly obtained. The goal is to define optimal sensors locations for known function of structural dynamic response and degrees-of-freedom. To determine the optimal subset of set of all potential nodes of the structure sensors locations a mixed integer linear programming optimization task is formulated. The fitness function used as objective function is based on computing of the information difference between node with and without sensor. The information from locations without sensors is calculated by means of linear interpolation of the neighboring sensors data. The proposed optimal sensor placement strategy works by dropping out the sensors with smallest information loss thus providing the real function of structural dynamic response to be the closest one to the function of structural dynamic response curve with all sensors present.

II. OPTIMAL SENSORS LOCATIONS BY COMBINATORIAL OPTIMIZATION

Adequate sensor placement plays a key role in the fields of system identification, structural control, damage detection and structural health monitoring of flexible structures. In recent years, interest has increased in the development of methods for determining an arrangement of sensors suitable for characterizing the dynamic behavior of a given structure.

The problem of determining the optimum sensors locations has received considerable attention recently. In structural vibration measurements, the locations of sensors can be determined by past experience and knowledge of a structure or by finite element analysis. The mode shapes of a structure have influences on the measurements. Usually, the structural dynamic characteristics are analyzed first before making the



Fig. 1. Deviation of the measurement accuracy.

measurements of the structural dynamic response. At the analysis phase, the structure could be simplified as a system with more degrees-of-freedom or as a lumped mass system. The goal is to have more nodal points for detailed data of structural responses and a part of these nodes could be used as sensors locations. In general the more sensors are used; the more detailed information of the structure can be obtained. However, the more number of sensors are used, the more instruments and workloads are required and in practice a fixed number of sensors should be located on the best structure positions.

Assume the mode shape matrix of a structure for p modes is [12]:

$$\Phi = [\phi_1, \phi_2, \phi_3, ..., \phi_p]. \tag{1}$$

A widely used approach to a best sensor location problem is the degree of similarity of the real mode shape and the measured mode shape. That is usually done by defining of a proper fitness functions.

In this paper a new type of fitness function is used to formulate combinatorial optimization task for optimal locations of given number of sensors. Let assume that the structure has n degrees of freedom or nodes where the sensors can be located and the goal is to choose the best m of them as sensors locations. The data for (n - m) locations (without sensors placement) could be obtained by linear interpolation of the two neighboring points data. If *i*-th node has no sensor, its data could be calculated taking into account the structural dynamic response curve from the vibration analysis. In this case, the deviation of the measurement accuracy Δ_i could be illustrated as shown in Fig. 1.

The basic idea is to drop out the (n - m) sensors with smallest information accuracy loss, i.e. the sensors locations with smallest absolute value of Δ_i . That idea is realized in a fitness function used as objective function in a combinatorial optimization problem:

$$\sum_{i=0}^{p} \sum_{j=2}^{n-1} x_j |\phi_j^i - \frac{\phi_{j-1}^i - \phi_{j+1}^i}{2}| \longrightarrow \max,$$
(2)

where $x_j \in \{0, 1\}$, i = 1, 2, ..., n are binary integer variables assigned for all sensors locations, ϕ_j^i is the value of the *i*-th mode shape of *j*-th node.

The requirement for finding of m best sensors locations is introduced by the constraint:

$$\sum_{i=1}^{n} x_j = m. \tag{3}$$

In that approach the first and last sensors locations nodes are always present, i.e.:

$$x_1 = 1, \tag{4}$$

$$x_{10} = 1.$$
 (5)

Dropping out the sensors with smallest Δ_i means that the function of structural dynamic response will be the closest one to the function with all sensors present. It is obvious that when the selection of sensor locations is optimum, then (2) should be maximized and the selection of the sensors locations is equivalent to solving a combinatorial optimization problem (2) subject to (3-5). The solution of the formulated optimization task defines m optimal sensors locations. It should be pointed out, that the measure (2) is applied simultaneously for all p mode shapes. That means to define a compromise optimal choice of sensors number to be dropped out, complying with all p mode shapes.

The determination of optimal placements of sensors is equivalent to solving an optimal problem as stated above. The optimal criterion for determining the sensor locations are based on the definition of the fitness function (2).

III. ALGORITHM FOR USING OF THE PROPOSED OPTIMIZATION APPROACH

With the rapid development of the micro-electro-mechanical systems technology, a future trend of maintenance research and developments would be the design of intelligent device which has the capability of continuously monitoring its own health using on-line data acquisition, on-line signal processing and on-line diagnostic tools [13].

Vibration analysis has been refined for todays industrial market to provide fast efficient measurements of rotating machinery vibration. Frequency spectrum monitoring is undoubtedly the key to diagnostics, respectively to predictive maintenance using vibration monitoring. Vibration based condition monitoring is the process in which the machine components are regularly checked and the condition i.e., whether it is healthy or faulty, is checked on the basis of vibration signals got from the machine components. Vibration monitoring can be broadly carried out at three levels [11]:

- overall vibration level measurement, to detect that a problem exists.

- spectral or frequency analysis, to locate where the problem is in the machine.

- special techniques, which can indicate what the problem is at a more detailed level.

The signal processing stage includes time domain data capture, digital decimation/filtering, windowing, fast Fourier transform (FFT) analysis, FFT averaging, and record storage (Fig. 2). The FFT is a highly efficient algorithm for obtaining the Fourier transform of discretized time signals. Various implementations of the FFT algorithm have become the workhorse of signal analysis packages and the FFT chips are incorporated in portable signal analyzers.



Fig. 2. Time waveform.



Fig. 3. Vibration alarm plot.

The speed of predictive maintenance depends of signal processing techniques and diagnostic methods. The plots of amplitude vs. time waveform and amplitude vs. frequency (FFT) are required for the trained technician or engineer to analyze and determine the machine fault. Since each rotating element generates an identifying frequency, analyzing the frequency disturbances will identify the faulty element. Once the fault is identified, parts can be ordered and repairs can be scheduled.

A recent development in the predictive maintenance and reliability market is to leverage the investment already made in process control systems like programmable logic controller, distributed control system, supervisory control and data acquisition. This allows the operations, maintenance, and process control teams to monitor and alarm vibration levels on critical machines. Using a standard output, the loop power vibration transmitters and sensors provide a current output proportional to the overall value of the machine vibration (Fig. 3) [12]. This is not a dynamic analog signal, and it cannot be used to analyze the machine fault, but it can be used to alert when the vibration levels are too high.

Predictive maintenance includes methods for data acquiring and information fusion combined with signal processing. As



Fig. 4. Algorithm for using of the proposed approach for predictive maintenance.

a result of signal processing fault detection possibilities can be recognized. Taking into account all of described above considerations, an algorithm for predictive maintenance using the proposed approach for defining of optimal sensors locations and signal processing is shown in Fig. 4.

First step of the algorithm involves defining of the structure properties and proper choice of sensor type. It is very important to choose the correct sensor, cable, connector, and mounting method for each application. This will provide quality measurements and accurate vibration data for identifying faults in machinery.

To select the proper sensor type the following parameters should be considered: measurement range (sensitivity), frequency range, broadband resolution (noise), temperature range, sensitivity tolerance and size. The defining of sensors locations on the second step can be done by past experience and/or knowledge of a structure and/or by using of finite element analysis. The proposed fitness function (2) is used to formulate mixed integer programming task on the third step. The task solution defines optimal sensors locations. On the next step the signals from the installed sensors are collected and processed. Intelligent processing of sensor data is one of the most important elements of structural health monitoring systems. The major elements of signal processing for damage detection include: data pre-processing, feature extraction and selection, pattern recognition and data/information fusion [11, 13].

The results from signal processing are used to fault classification. If the fault classification is impossible or wrong the optimization task should be modified and solved again (Fig. 4). After successful fault determination a proper decision

TABLE IThe First Three Mode Shapes of Structure

Node	Mode 1	Mode 2	Mode 3
1	0.15643	0.45400	-0.70706
2	0.30902	0.80901	-0.99993
3	0.45400	0.98770	-0.70706
4	0.58779	0.95107	0.00004
5	0.70711	0.70711	0.70701
6	0.80901	0.30902	1.00000
7	0.89103	-0.15643	0.70704
8	0.95107	-0.58779	0.00005
9	0.98770	-0.89103	-0.70713
10	1.00000	-1.00000	-0.99986



Fig. 5. Dynamic response functions from Table 1 (10 sensors).

and actions are taken including recommendations for efficient maintenance policies. Making a decision implies that there are alternative choices to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that has the highest probability of success or effectiveness and best fits with the goals.

IV. NUMERICAL EXAMPLE FOR DETERMINATION OF OPTIMAL SENSORS LOCATIONS

A numerical example based on known data for vibration displacement of a cantilever structure is used. The first three mode shapes calculated from free vibration analysis are shown in Table 1 [14] and Fig. 5. Every mode shape is normalized to have maximum value of each mode shape equal to 1.

The structure is divided into 10 equal sections with node number 10 located at the free end of the cantilever.

The mixed integer linear programming formulation problem with objective function (2) subject to (3-5) is used for 4 practical examples. The goal of these 4 cases is to select 9, 8, 7 and 6 optimal sensors locations (m = 9, 8, 7, 6) among the 10 node locations. The formulated optimization tasks are solved by means of LINGO solver [15] implementing branch and bound method. The results of the optimization tasks solutions are shown in Table 2 and are illustrated in Fig. 6.

The results from numerical experiments demonstrate that it is possible to use the proposed approach for optimal placements of fixed number of sensors. The optimal solutions define sensors to be removed as best compromise for all mode shapes simultaneously. The task with 9 sensors locations (Fig. 6a) is

TABLE II Results of Tasks Solutions

Task	Sensors	Removed	Mode 1	Mode 2	Mode 3
	to locate, m	sensors	$\sum \Delta_i$	$\sum \Delta_i$	$\sum \Delta_i$
1	9	4	0.007235	0.10366	0.000065
2	8	4,7	0.018225	0.12071	0.207080
3	7	4, 5, 7	0.026935	0.197775	0.414070
4	6	4, 5, 7, 9	0.039100	0.294910	0.621295

better approximation to the original response function curves then task with 8 sensors locations (Fig. 6b) etc.

The summarized difference of the measurement accuracy Δ_i (the last 3 columns in Table 2) as a result of sensor removing is increased. The last task shows that the choice of sensors to be removed is compromise between mode shapes for some mode shapes the curve approximation is not too good (Mode 3). The signal processing results are to be used for determining of the number of the sensors to be removed in such way the quality measurements and accurate vibration data for identifying faults is still adequate.

The numerical results show the applicability of the proposed approach for determination of optimal sensors locations. The location of reduced sensors depends on the summarized deviation in the structure response functions. It should be noted that all dynamic response functions are considered simultaneously to define the deviation for each two neighbor sensors locations.

V. CONCLUSION

Taking the advantage of predictive maintenance many unexpected problems could be avoided by improving the general maintenance level of system. Vibration analysis has been used as a predictive maintenance procedure and as a support for machinery maintenance decisions. As a general rule, machines don't breakdown or fail without some form of warning, which is indicated by an increased vibration level. By measuring and analyzing the machine's vibration, it is possible to determine both the nature and severity of the defect, and hence predict the machines failure. The paper discusses predictive maintenance of structures relies essentially on optimal sensors location. The optimal sensor location problem is to estimate the required number of sensors for the best sensor performance and to find the best positions of sensors for a specific task. The paper addresses optimal sensors locations problem by using of combinatorial optimization modeling technique to formulate mixed integer linear programming tasks. The tasks solutions define optimal sensors locations for a given number of sensors. The results of chosen sensors locations give as close as possible repeating the curve of structure dynamic response function. The used fitness function is based on the difference between sensor information if sensor is present and information calculated by linear interpolation if sensor is not present. The proposed approach is implemented in an algorithm for predictive maintenance and numerical results indicate that it is suitable for practical application. It could be used for determining of information loss for different number of sensors. The signal processing and the fault defection results could be used for determining of the proper sensors number



Fig. 6. Dynamic response functions with: a) 9 sensors, b) 8 sensors, c) 7 sensors, d) 6 sensors.

in such way the quality measurements and accurate vibration data for identifying faults is still adequate.

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