INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2016, VOL. 62, NO. 4, PP. 379-388
Manuscript received October 4, 2016; revised November, 2016. DOI: 10.1515/eletel-2016-0052

A Minimum-Spanning-Tree-Inspired Algorithm for Channel Assignment in 802.11 Networks

Iwona Dolińska, Mariusz Jakubowski, Antoni Masiukiewicz, Grzegorz Rządkowski, and Kamil Piórczyński

***Abstract*— Channel assignment in 2.4 GHz band of 802.11 standard is still important issue as a lot of 2.4 GHz devices are in use. This band offers only three non-overlapping channels, so in crowded environment users can suffer from high interference level. In this paper, a greedy algorithm inspired by the Prim's algorithm for finding minimum spanning trees (MSTs) in undirected graphs is considered for channel assignment in this type of networks. The proposed solution tested for example network distributions achieves results close to the exhaustive approach and is, in many cases, several orders of magnitude faster.**

***Keywords*— channel selection, channel assignment, 802.11 home networks, greedy algorithm**

I. INTRODUCTION

THE interference issue is still present in 2.4 GHz band of 802.11standard. Since a few releases of 802.11 are in use (b/g/n/ac), it is necessary to secure reliable transmission in this band. The number of non-overlapping channels is only three and in environments with a high density of access points this number is not sufficient. Very often the channel assignment is quite chaotic and as a results we obtain a combination of three 1, 6, 11 channels suggested as the best choice by Internet Service Providers together with other channels which are selected individually by users basing on their own criteria. Typical channel distribution registered in a seven-room house is shown in Fig. 1[24]. The number of access points (APs) in each room was established using Xirrus Wi-Fi Inspector software [6].

The number of APs detected in each room is different and varies from 11 to 28 and the number of APs transmitting on the same channel varies from 0 to 17. Most occupied channels are 1, 6, and 11, respectively.

The authors also tested two real networks in the previous work [7] and made an optimization of channel selection using tools which were available for ordinary user. The results showed total lack of any channel coordination in private networks and necessity of future works to improve the situation. There are some methods and software tools for improving channel allocation. A survey of methods is presented in the next section. These methods are usually too difficult for ordinary users and the optimization is typically carried out from the point of view of a single network, so the results can be temporary only.

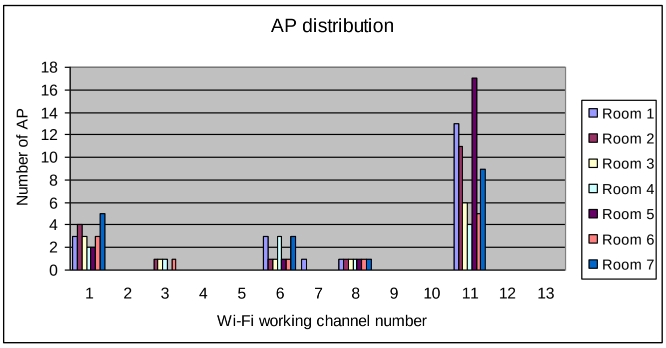


Fig 1. Typical distribution of channel assignment to APs in densely populated environment.

There is also a dynamic solution [4] but the chipsets which are produced currently do not guarantee sufficiently fast switching time, so a lot of time can be wasted just for switching if many APs start optimization at the same time.

The authors proposed previously [22] two algorithms for static channel allocation. The first method is based on modulo 3 mechanism. It is assumed that network boundaries adhere to each other and that only three non-overlapping channels are the subject of assignment. Subsequently, this method is used for building a reference array for the RAA (Reference Array Algorithm) algorithm. The RAA algorithm can be used when APs are spread over limited area.

In this paper, the authors propose and test an algorithm, which improves RAA in terms of achieving a lower interference level between networks. The algorithm is inspired by the Prim's algorithm for finding minimum spanning trees (MSTs) in undirected graphs [23]. In the proposed approach, a set of Wi-Fi APs is modeled as a weighted undirected graph in which weight values are proportional to the interference level between APs. To this end, a simple measure is proposed which takes into account both the physical and the channel distance between the APs. Thus, the weights of the graph dynamically change with channel numbers assigned to particular APs (graph vertices). The optimization criterion is to minimize the total amount of interference within the set of APs. Similarity to the Prim's algorithm stems from the fact that the optimization is carried out locally i.e., only direct neighborhood of a given node (AP) is taken into account. Typically, the algorithm obtains close-to-optimal results within a fraction of time required by the exhaustive approach which performs global optimization.

The rest of the paper is organized as follows. In Section 2, an overview of available methods for channel allocation is presented. Section 3 gives a general overview of adjacent channel interference issue. In Section 4, assumptions for algorithms are presented and the proposed algorithm is described. Simulation results are presented in Section 5 and the conclusions are given in Section 6.

All the Authors are with Faculty of Engineering, Vistula University, ul. Stokłosy 3, 02-787 Warszawa, Poland (e-mail: i.dolinska@vistula.edu.pl, m.jakubowski@vistula.edu.pl, a.masiukiewicz@vistula.edu.pl, grzerzad@gmail.com, kamilrufi1992@op.pl).

II. RELATED WORKS

There are a lot of different types of 802.11 networks. Among them administrated networks and small home or private networks. The first ones are usually planned and optimized, while the second ones are rather chaotic. There are several methods of Wi-Fi network planning described in the literature e.g., Nelder-Mead direct planning [5], special planning software tools for high density locations such as conference rooms, big halls etc. [6]. Integrated management methods are used in administrated networks. Such methods based on balanced throughput distribution which guarantees a required fairness level [8]. Next solution employs the dynamic power level control [9, 10]. The channel assignment method can help to achieve many goals such as: the best network capacity, the minimal interference power level, the maximal throughput or the fairness guaranteed. Typical targets for planned and administrated networks are the maximal network capacity and the minimal interference power level. The visualization method of measurement results is the map coloring [11] which supports administrator decisions.

Different attitude to this problem was proposed by Choi et al. 2002 [12]. With Integer Linear Programming (ILP) two parameters are analyzed: the distribution of APs and the data amount distribution among APs. Algorithm requires a lot of information concerning the network and intensive calculation and switching. Priority map is the solution of Wertz et al. 2004 [13]. The method is quite complicated and a few steps are necessary to obtain an optimal network. Some modification of AP number and location might be applied. Patching algorithm [14] includes the fairness issue. CFAssign-RaC [15] analyzes 802.11 environment from the point of view of both APs and users. The optimization includes the channel assignment and the throughput balance.

Small private networks suffer very often since the optimization of channel selection is difficult [7]. Three methods are dedicated for small home networks. LCCS [16] analyzes the number of users per channel, while Pick Rand [17] calculates the coefficients of superposed channels. SAW [4] is a dynamic method basing on continuous interference calculations and channel switching to find the momentary best solution.

The discussed concepts presented in literature are not well suited for chaotic Wi-Fi home networks environment. Some of them are too complicated, some offer dynamic solutions with high switching time consumption, so the authors decided to look for a static method which could be quite easily implemented for both existed and new Wi-Fi 2.4 GHz networks.

The first of previously developed methods [22] is based on modulo 3 mechanism. It is assumed that network boundaries adhere to each other and that only three non-overlapping channels are the subject of assignment; however, it may be advisable to make similar analysis for more channels. Subsequently, the first method is used for building a reference array for the RAA algorithm. The RAA algorithm can be used when access points (APs) are spread over limited area. Both methods are based on three main principles: we can assign a unique number to each AP, we know the X, Y coordinates of each AP, and we use only three channels 1, 6, and 11. The proposed Modulo 3 algorithm is limited to simple cases when AP coverage areas adhere to each other, so practically is used only for building the reference array. RAA enables channel assignment in networks with random AP location, so it can be used for real networks. RAA does not solve border conditions and three-dimension arrangements which we have in blocks of flats. The minimal distance between the same channels equal to d/2 is guaranteed by the RAA mechanism, where the d value corresponds to the coverage of a single AP. However, the RAA algorithm does not guarantee optimal channel assignment in the sense that the amount of interference between APs is minimized. Besides, it is limited only to two-dimensional cases.

III. INTER CHANNEL INTERFERENCE IN 802.11 NETWORKS

The signal strength of 802.11 standard in each channel is limited by the spectral filter, however, these filters overlap partly [3]. The correlation between the signal power in different channels can be described by the Correlation Ratio Coefficient (CRC). For two stations belonging to different networks which transmit in the same channel the CRC is 1 or 100%, which means that the only differentiation is the signal level. The CRC can be calculated in a few ways. For instance, as the overlapping area under the mask curves or the overlap of frequency ranges. Another method, which is used in this paper, considers the signal power density per Hz and calculates the overlapping of frequency ranges of different channels. The results significantly depend on the bandwidths which are taken into account [7].

The carriers or sub-carriers are located within the 20 MHz channel. While we consider the correlation between channels of 20 MHz wide, the CRC values are similar to those calculated for 20 MHz power density distribution. The characteristic of the power distribution is the equivalent to a part of the spectral mask (see the Fig. 2). The CRC calculation results are presented in Table I.

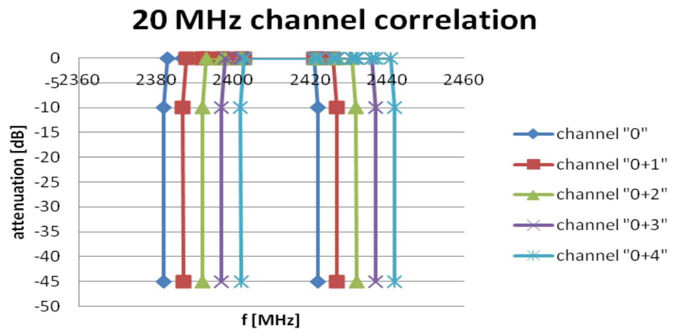


Fig 2. CRC for 20 MHz channel width, when the correlation is calculated as the area of the spectral mask characteristic within 20 MHz band.

To calculate the interference power within a given channel, the following formula is used:

$$P_{N-INT} = \sum_{k=1}^{11}\sum_{x=1}^{m} P_{kx} \cdot CRC_z, \tag{1}$$

where $P_{N\text{-}INT}$ represents the interference power in a given channel. It is the sum of the power of all stations detected in all channels (P_{kx}) after we take into account the correlation coefficient (CRC_z), when k is a current channel, z is the channel interval $z = |N - k,|$ where N is a given channel and where x represent all other stations in each channel.

TABLE I
CORRELATION RATIO COEFFICIENT (CRC) VALUES

Channel interval	CRC [%]
0	100
1	75
2	50
3	30
4	0
5	0
6	0
7	0
8	0
9	0
10	0

IV. PROPOSED ALGORITHM DESCRIPTION

A. *Preliminary assumptions*

Before the algorithm development, the following assumptions are made:

a) 2.4 GHz band is used and three channels 1, 6, and 11 or four channels 1, 4, 7, and 11 are considered,
b) each AP has its own unique number,
c) the environment is static i.e., the number of APs is fixed and does not change
d) coordinates (x, y, z) of each AP within the considered area are known,
e) the result of the channel assignment is the same regardless by which AP it is performed i.e., all APs should get exactly the same channel assignment,
f) new AP can be incorporated into the existing structure just by assigning a proper channel number.

The b) assumption is easily met since each AP has its own unique MAC address. MAC address is in fact a number written in hexadecimal code. We can arrange these addresses in the ascending order and assign 1 to the lowest number and N to the highest number, where N is the number of APs inside the analyzed area.

The d) assumption, in principle, can be also met. At first, we need the location of our own AP, then the method of distributing this information and finally, we have to collect data about other AP locations. The authors considered the possible solution in the previous paper [18]. Nevertheless, it is worth to mention that a few methods can be useful such as GPS, digital maps or tools implemented in 802.11 v and k amendments.

The target is to achieve a unique channel assignment when three channels 1, 6, and 11 or four channels 1, 4, 7, and 11 are available.

B. *Algorithm description*

1) *Modeling a set of APs as a graph*

First, let us consider a set of five Wi-Fi APs as depicted in Fig. 3. Between each AP pair (k, n), interference occurs with strength $F_{k,n} = F_{n,k}$ which depends both on the distance in the physical and channel space. In particular, when physical separation between two APs increases, the strength of their interaction decreases. Similarly, when separation in the channel space increases i.e., the difference between channel numbers on which each AP transmits, the strength of interaction decreases. Thus, a set of Wi-Fi APs can be modeled as a complete graph i.e., a simple undirected graph in which every pair of distinct vertices is connected by a unique edge, with weights.

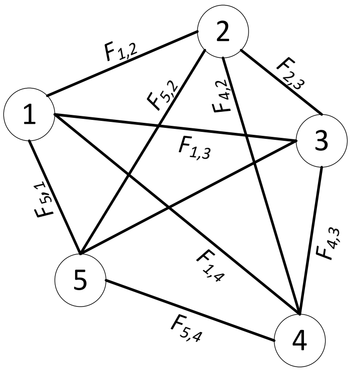


Fig 3. Set of APs as a complete weighted graph.

In this paper, values of weights $F_{k,n}$ are calculated according to formula:

$$F_{k,n} = \frac{\delta(C_k, C_n)}{L^2}, \tag{2}$$

where $\delta(C_k, C_n)$ is the CRC value according to Table I, C_k is the channel number assigned to AP_k and C_n to AP_n, and L is Euclidean distance between APs in three-dimensional space:

$$L = \sqrt{(x_k - x_n)^2 + (y_k - y_n)^2 + (z_k - z_n)^2}. \tag{3}$$

Since the received RF power is inversely proportional to the square of the distance from the source, L^2 was put in the denominator. It is noteworthy that for $|C_k - C_n| > 3$, $\delta(C_k, C_n) = 0$ and thus, $F_{k,n}$ is equal to zero regardless of the physical distance between the APs.

This measure does not include attenuating effects of the building structure and other obstacles, which should be considered in future investigations.

Regarding the task of optimal channel assignment to APs, its primary goal is to minimize the total level of interference between APs, which can be expressed as the sum of interactions between every pair of APs:

$$F_{tot} = \sum_{k=1}^{N-1} \sum_{n=k+1}^{N} F_{k,n}, \tag{4}$$

where N is the number of APs in the set. However, the same value of F_{tot} can be obtained for different sets of $F_{k,n}$ values. For instance, for a given channel distribution and a specific pair of APs, $F_{k,n}$ value might be particularly large. On the other hand, for another channel distribution and the same value of F_{tot}, this value might be smaller at the expense of some increase of interferences between the remaining APs. Since we are interested not only in small value of F_{tot} but also in even distribution of $F_{k,n}$ values, it would be beneficial to check how efficiently a given algorithm assigns channels from that point of view. To this end, for each AP_k, a critical value F_{c_k} is defined as:

$$F_{c\_k} = \frac{0.3}{L^2_{k\_min}}, \tag{5}$$

where 0.3 is the CRC value for the channel distance equal to 3 (see Table I) and L_{k_min} is the minimum distance between AP no. k and the remaining APs. For a given assigning algorithm, it is desirable that $F_{k,n}$ is smaller than F_{c_k} for every AP$_k$. In other words, $F_{k,n}$ should be equal to zero for all the APs within the distance of L_{k_min}. Note that it might be not always possible since several APs can lie in the distance equal to L_{k_min}. Nevertheless, information about exceeding of F_{c_k} can be used as an additional indicator of the algorithm efficiency.

2) Exhaustive procedure for channel assignment

A straightforward approach to the channel assignment problem can be the exhaustive testing of all possible combinations of channels assigned to APs, starting from C_{1_min}, C_{2_min}, ..., C_{N_min} to C_{1_max}, C_{2_max}, ..., C_{N_max}, where C_{min} and C_{max} represent the lowest and the highest channel number available and N is the number of APs in the set. For each such combination, $F_{k,n}$ values are first calculated pairwise according to (2) and then, they are summed up to obtain the F_{tot} value. The major advantage of such approach is that optimal assignment of channels will always be found. However, considering that the set of available channels and the set of APs contain M and N elements, respectively, all N-element variations with repetitions of the M-element set have to be tested. Thus, the number of possible assignments is M^N. It implies the exponential complexity of the exhaustive procedure, which makes it impractical in most applications in spite of its optimality.

3) Greedy procedure inspired by the Prim's minimum spanning tree algorithm

A minimum spanning tree (MST) of an undirected and connected edge-weighted graph is a tree which connects all the vertices of the graph for which the cost (sum of weights of all the edges in the tree) is minimal among all the spanning trees [23]. In the Prim's algorithm [23], the MST is built starting from an arbitrarily selected vertex and in each step, the minimum-weight edge is attached from the edges that connect the tree to vertices not yet included in the tree. In Fig. 4, an example graph with its MST found with the Prim's algorithm is shown. First, starting from the vertex no. 1, the edge with weight equal to 1 is selected which connects vertex no. 5. Next, the edge connecting vertex no. 3 is selected as the cheapest one (weight equal to 4). Subsequently, edges connecting vertices no. 4 and 2 are added.

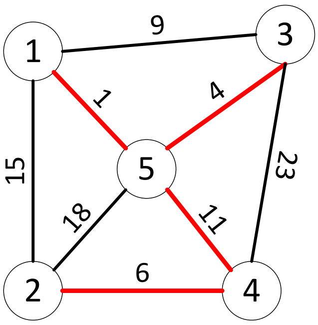


Fig 4. Weighted graph example with edges belonging to the MST indicated by thicker red lines.

Obviously, the problem of assigning channel numbers to Wi-Fi APs is not equivalent to finding the MST. In particular, the tree encompasses the whole graph and is static, and the objective is to search for a set of weights which gives minimum cost. Nevertheless, a similar greedy approach can be applied to assigning channels (weights) to APs. First, to an arbitrarily chosen AP e.g., no. 1, the lowest channel number available C_{min} is assigned. Next, an AP is found which is the closest to the previously visited one (if there is more than one within the same distance, the one with the lowest number can be selected), and to this AP, the lowest channel number is assigned which minimizes the current total cost F_{tot_c}, which at this point is simply equal to the strength of interaction between the two APs. Next, the previous step is repeated i.e., the search for the closest neighbor of the previously visited AP which does not have a channel number assigned yet, and the lowest possible channel number is assigned according to the same rule – to minimize the up-to-date total cost. This is done by testing all available channel numbers and calculating F_{tot_c}. The process is continued up to the point when all the APs will get a channel number assigned (see Fig. 5). Using this cumulative approach, a final channel assignment is obtained. Since optimization is carried out locally, a great reduction of computational complexity is achieved with respect to the exhaustive procedure. In fact, instead of the exponential complexity, the quadratic complexity is obtained, which is determined by the cost of generating the squared distance matrix. Since this matrix is symmetric with zeros on the main diagonal, the number of operations necessary to generate its elements changes with N as $N\times[(N-1)/2]$. If the matrix were given, the complexity would increase linearly with N. The speed-up comes with a price however, as a potential loss of optimality of the solution can be expected.

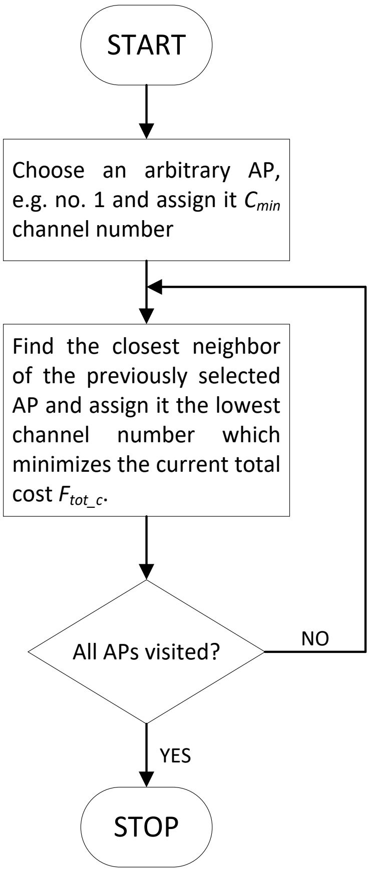


Fig 5. Flowchart of the greedy channel assignment procedure.

Let us analyze an example which will allow both to better explain the principle of working of the algorithm and to illustrate the potential risk of results deterioration. In Fig. 6, a set of eight APs on the plane is shown and its distance matrix is given in Table II. The distances are expressed in some arbitrary units. This arrangement looks artificial; however, it possesses certain interesting properties. Namely, all APs are almost in the same distance from AP no. 1 and APs in pairs (2, 3), (4, 5), and (7, 6) lie very close to each other (L = 0.1). Three channels are used in the example: 1, 6, and 11. The algorithm starts from AP no. 1 and assigns it channel 1. Since four neighboring APs are located in the same distance (no. 2, 4, 6, and 8), AP no. 2 is selected with channel 6 assigned.

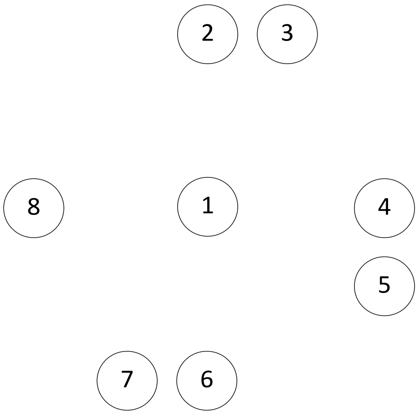
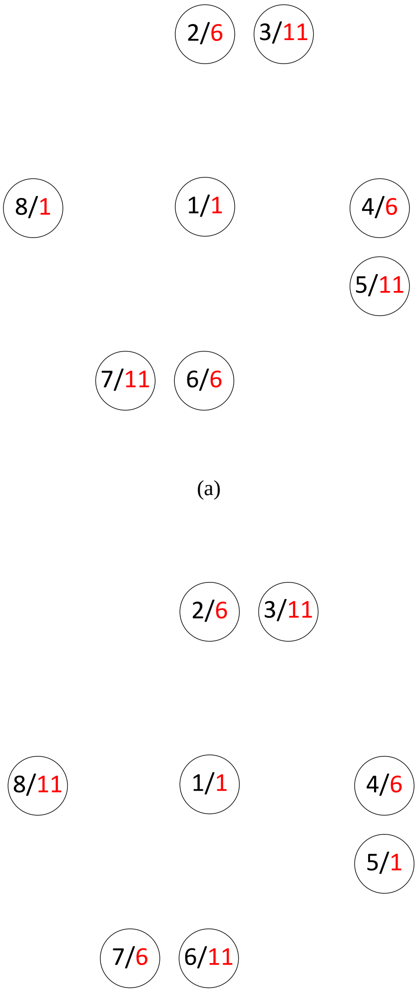


Fig 6. Set of APs on the plane.

TABLE II
DISTANCE MATRIX FOR APS IN FIG. 6

AP no.	1	2	3	4	5	6	7	8
1	0	1.000	1.005	1.000	1.005	1.000	1.005	1.000
2	1.000	0	0.100	1.414	1.486	2.000	2.002	1.414
3	1.005	0.100	0	1.345	1.421	2.002	2.010	1.486
4	1.000	1.414	1.345	0	0.100	1.414	1.486	2.000
5	1.005	1.486	1.421	0.100	0	1.345	1.421	2.002
6	1.000	2.000	2.002	1.414	1.345	0	0.100	1.414
7	1.005	2.002	2.010	1.486	1.421	0.100	0	1.345
8	1.000	1.414	1.486	2.000	2.002	1.414	1.345	0

At this point, F_{tot_c} is equal to zero, as channels 1 and 6 do not interfere witch each other. AP closest to no. 2 is no. 3, so it obtains channel 11 to maintain F_{tot_c} equal to zero. From AP no. 3, the algorithm goes to AP no. 4, since no. 1 was already visited. The algorithm calculates F_{tot_c} for all three available channels and assigns channel 6 to AP no. 4. Since the distance between AP no. 4 and AP no. 2, which also has channel 6 assigned, is the largest one, this choice is the best one possible. Next, the algorithm proceeds with APs no. 5, 6, 7, and 8, and assigns them channels 11, 6, 11, and 1, respectively. For AP no. 8, channel 1 is selected in spite of the fact that its closest neighbor AP no. 1 has the same channel number. However, since already three APs (no. 2, 4, and, 6) have assigned channel 6 and other three APs (no. 3, 5, and 7) have assigned channel 11, the total amount of interaction between APs will be smallest with channel 1 assigned to AP no. 8. This is an example of unevenness of local interactions, when the total level of interaction between APs takes priority. Final channel assignment generated by the greedy algorithm is {1, 6, 11, 6, 11, 6, 11, 1} as shown in Fig. 7 (a) and the F_{tot} value for the set of APs is equal to 3.488. Analysis of local interaction values provides an additional insight into the assignment quality. Comparison with critical values calculated for each AP shows that for APs no. 1 and 8, the F_{c_k} value is exceeded. According to (5), the F_{c_k} value for these APs is equal to 0.3 (L_{1_min} = L_{8_min} = 1), while the strength of interaction between them is equal to 1 ($\delta(1, 1) = 1$). The assignment generated by the exhaustive procedure shown in Fig. 7 (b) is {1, 6, 11, 6, 1, 11, 6, 11} with the F_{tot} value equal to 3.394, which is less than 3% lower than in the case of the greedy procedure. Additionally, since channel 1 is assigned to AP no. 5, the critical value is not exceeded for AP no. 8 but still excided for AP no. 1. However, this improvement is obtained at the expense of about 16-times processing time consumption increase. Considering that the algorithm is designed to run on low-end microprocessors, the small improvement does not seem to be worth additional burden. In the next section, several other distributions of APs in two- and three-dimensional space are analyzed, sometimes with much more dramatic differences in terms of the computational power consumption.



(a)

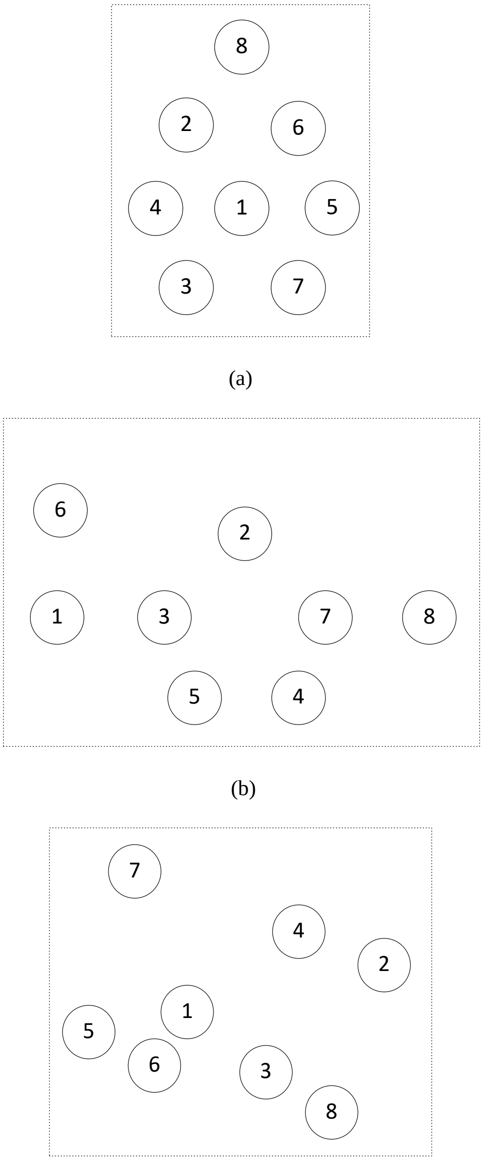
(b)

Fig. 7. Channel assignment to APs from Fig. 6: (a) by the greedy procedure, (b) by the exhaustive procedure.

V. SIMULATION RESULTS

In this section, the greedy procedure, described previously, is tested against the exhaustive one for several sets of APs both on the plane and in three-dimensional space. In particular, three sets of eight APs distributed on the plane and three sets of 16 APs distributed on two levels are considered with two sets of available channels: {1, 6, 11} and {1, 4, 7, 11}. Channels in the first set do not interfere with each other; however, they provide fewer degrees of freedom and can introduce high local interference level when the same channels are assigned to two closely located APs (see the example in the previous section). On the other hand, some channels in the second set interfere with each other; however, since the degree of interference is relatively small, they can reduce amount of interference in comparison with the three-channel set. It might be especially useful for three-dimensional distributions.

The goal of the experiments is to assess the greedy algorithm performance. The figures of merit are: algorithm execution time (t_{ex}) and total strength of interaction within the set (F_{tot}). Additionally, information whether local interactions are kept below their critical values F_{c_k} is collected. Three AP distributions on the plane are shown in Fig. 8 and their distance matrices are presented in Table III, IV, and V. They can be characterized as regular, quite regular, and random. These distributions are referred throughout the paper as 2D-I, 2D-II and 2D-III, respectively. Three-dimensional distributions are created by duplicating a two-dimensional distribution at each of two levels which are vertically separated by 0.2 arbitrary unit. It means that the first three-dimensional distribution, referred as 3D-I, is created by duplicating distribution 2D-I at the level 0 and 1 i.e., AP no. 1 at the level 1 (denoted as 1_1) is placed directly 0.2 unit above AP no. 1 at the level 0 (denoted as 1_0) and so on. Distributions 3D-II and 3D-III are created in analogical way from the distributions 2D-II and 2D-III. Placing APs directly one above the other creates the most demanding conditions for an assigning algorithm and is also not quite unreasonable as in a block of flats APs are typically located at similar points such as living rooms or antechambers. Three-dimensional distributions of APs can be found in Fig. 9. Since distances between APs at the same level are exactly the same as for their parental two-dimensional distributions, only distances between APs at different levels are shown in Table VI, VII, and VIII.



(a)

(b)

(c)

Fig. 9. AP distributions on the plane: (a) 2D-I, (b) 2D-II, (c) 2D-III.

TABLE III
DISTANCE MATRIX FOR 2D-I DISTRIBUTION

AP no.	1	2	3	4	5	6	7	8
1	0	1.003	1.003	1.000	1.000	1.003	1.003	1.740
2	1.003	0	1.740	1.003	1.734	1.000	2.006	1.003
3	1.003	1.740	0	1.003	1.734	2.006	1.000	2.657
4	1.000	1.003	1.003	0	2.000	1.734	1.734	2.006
5	1.000	1.734	1.734	2.000	0	1.003	1.003	2.006
6	1.003	1.000	2.006	1.734	1.003	0	1.740	1.003
7	1.003	2.006	1.000	1.734	1.003	1.740	0	2.657
8	1.740	1.003	2.657	2.006	2.006	1.003	2.657	0

TABLE IV
DISTANCE MATRIX FOR 2D-II DISTRIBUTION

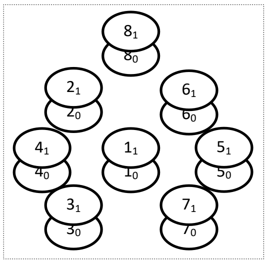
AP no.	1	2	3	4	5	6	7	8
1	0	2.283	1.000	2.585	1.755	1.580	2.580	3.580
2	2.283	0	1.628	2.661	2.653	1.786	1.638	2.298
3	1.000	1.628	0	1.755	1.217	1.869	1.580	2.580
4	2.585	2.661	1.755	0	1.000	3.592	1.212	1.740
5	1.755	2.653	1.217	1.000	0	3.050	1.740	2.567
6	1.580	1.786	1.869	3.592	3.050	0	3.025	3.913
7	2.580	1.638	1.580	1.212	1.740	3.025	0	1.000
8	3.580	2.298	2.580	1.740	2.567	3.913	1.000	0

TABLE V
DISTANCE MATRIX FOR 2D-III DISTRIBUTION

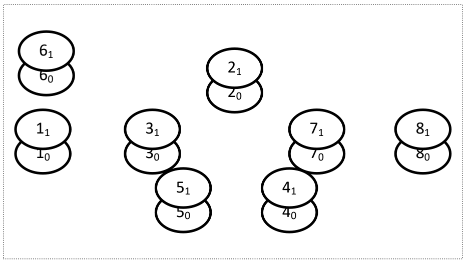
AP no.	1	2	3	4	5	6	7	8
1	0	3.788	1.643	2.816	1.960	1.115	2.889	3.068
2	3.788	0	3.064	1.514	5.694	4.865	4.879	3.152
3	1.643	3.064	0	2.801	3.434	2.292	4.338	1.425
4	2.816	1.514	2.801	0	4.535	3.929	3.371	3.546
5	1.960	5.694	3.434	4.535	0	1.203	2.898	4.783
6	1.115	4.865	2.292	3.929	1.203	0	3.341	3.596
7	2.889	4.879	4.338	3.371	2.898	3.341	0	5.703
8	3.068	3.152	1.425	3.546	4.783	3.596	5.703	0

TABLE VI
DISTANCES BETWEEN LEVEL 0 AND 1 APS FOR 3D-I DISTRIBUTION

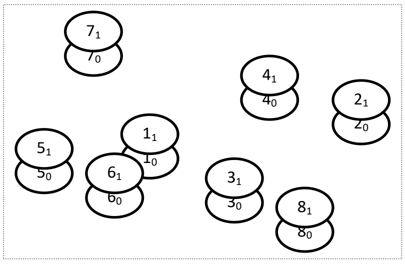
AP no.	1_1	2_1	3_1	4_1	5_1	6_1	7_1	8_1
1_0	0.200	1.023	1.023	1.019	1.019	1.023	1.023	1.751
2_0	1.023	0.200	1.751	1.023	1.745	1.019	2.016	1.023
3_0	1.023	1.751	0.200	1.023	1.745	2.016	1.019	2.665
4_0	1.019	1.023	1.023	0.200	2.010	1.745	1.745	2.016
5_0	1.019	1.745	1.745	2.010	0.200	1.023	1.023	2.016
6_0	1.023	1.019	2.016	1.745	1.023	0.200	1.751	1.023
7_0	1.023	2.016	1.019	1.745	1.023	1.751	0.200	2.665
8_0	1.751	1.023	2.665	2.016	2.016	1.023	2.665	0.200



(a)



(b)



(c)

Fig. 10. Three-dimensional AP distributions: (a) 3D-I, (b) 3D-II, (c) 3D-III.

TABLE VII
DISTANCES BETWEEN LEVEL 0 AND 1 APS FOR 3D-II DISTRIBUTION

AP no.	1_1	2_1	3_1	4_1	5_1	6_1	7_1	8_1
1_0	0.200	2.292	1.019	2.592	1.767	1.592	2.587	3.585
2_0	2.292	0.200	1.641	2.668	2.661	1.797	1.650	2.307
3_0	1.019	1.641	0.200	1.767	1.233	1.880	1.592	2.587
4_0	2.592	2.668	1.767	0.200	1.019	3.598	1.229	1.752
5_0	1.767	2.661	1.233	1.019	0.200	3.057	1.752	2.575
6_0	1.592	1.797	1.880	3.598	3.057	0.200	3.032	3.918
7_0	2.587	1.650	1.592	1.229	1.752	3.032	0.200	1.019
8_0	3.585	2.307	2.587	1.752	2.575	3.918	1.019	0.200

TABLE VIII
DISTANCES BETWEEN LEVEL 0 AND 1 APS FOR 3D-III DISTRIBUTION

AP no.	1_1	2_1	3_1	4_1	5_1	6_1	7_1	8_1
1_0	0.200	3.793	1.655	2.823	1.970	1.133	2.896	3.074
2_0	3.793	0.200	3.071	1.527	5.697	4.869	4.884	3.158
3_0	1.655	3.071	0.200	2.808	3.440	2.300	4.343	1.439
4_0	2.823	1.527	2.808	0.200	4.540	3.934	3.377	3.551
5_0	1.970	5.697	3.440	4.540	0.200	1.219	2.905	4.787
6_0	1.133	4.869	2.300	3.934	1.219	0.200	3.347	3.602
7_0	2.896	4.884	4.343	3.377	2.905	3.347	0.200	5.707
8_0	3.074	3.158	1.439	3.551	4.787	3.602	5.707	0.200

The experiments are conducted using C++ implementation of both algorithms run on Intel Core i5 3.5 GHz CPU. The results concerning t_{ex} and F_{tot} are gathered in Table IX, X, XI, and XII. Information about local-level interactions between APs is presented in Table XIII for the exhaustive algorithm and in Table XIV for the greedy one.

For the execution time measurement, C++ clock() function is used, which is not considered accurate enough for measurement of short time periods. Hence, t_{ex} for the greedy algorithm is indicated only as below 1 ms, as the value returned by the function is typically 0.000. Nevertheless, the exponential increase of t_{ex} for the exhaustive algorithm can be easily observed. In particular, finding the optimal solution for 16 APs on two levels with four available channels takes well over six hours. On the other hand, for the greedy algorithm, the number of APs or channels does not influence the execution time sufficiently to register the change; however, some increase obviously occurs.

Comparing F_{tot} values found for each distribution, it can be noted that they decrease as the AP distributions are getting sparser. It is not surprising since with larger physical distances between APs, the level of interference decreases. Also the usage of four instead of three channels turns out to be beneficial for the F_{tot} reduction. In the case of the 3D-III distribution, even over 14-percent reduction of F_{tot} is observed when the exhaustive algorithm is used. However, the greedy algorithm is clearly not as efficient as the exhaustive one in utilizing more channels. For the 2D-II distribution, a 5-percent deterioration of the result is observed and for the 3D-I distribution, the improvement is less than 1%.

Comparing F_{tot} values obtained by each of the algorithms, it can be seen that discrepancies are, in general, below 10% with an exception of the 2D-II and 3D-I distributions with four channels, for which the difference slightly exceeds 10%.

Regarding the level of local interactions between APs i.e., whether the critical value is achieved or exceeded for a given AP, it can be seen that for both algorithms such cases are registered; however, only for 2D-I and 2D-II distributions both with three and four channels. Differences between the algorithms are not significant as they perform similarly on this aspect. Since avoiding the F_{c_k} violation is not specified as an optimization criterion but treated merely as information, it can be only said that none of the algorithms have an intrinsic protection against such cases. However, if the more even level of local interactions takes priority, modifications might be introduced leading to a channel assignment possibly with a higher F_{tot} value.

TABLE IX

RESULTS FOR TWO-DIMENSIONAL DISTRIBUTIONS WITH THREE CHANNELS

Distribution	Algorithm	Channel assignment: {AP1, AP2, AP3, AP4, AP5, AP6, AP7, AP8}								t_{ex} [s]	F_{tot}
2D-I	Exhaustive	1	6	6	11	6	11	11	1	0.021	2.321
	Greedy	1	11	11	6	11	6	6	1	<0.001	2.321
2D-II	Exhaustive	1	1	6	1	11	11	11	6	0.025	1.179
	Greedy	1	1	6	1	11	11	11	6	<0.001	1.179
2D-III	Exhaustive	1	1	6	6	6	11	11	11	0.024	0.528
	Greedy	1	1	11	11	11	6	6	6	<0.001	0.528

TABLE X

RESULTS FOR TWO-DIMENSIONAL DISTRIBUTIONS WITH FOUR CHANNELS

Distribution	Algorithm	Channel assignment: {AP1, AP2, AP3, AP4, AP5, AP6, AP7, AP8}								t_{ex} [s]	F_{tot}
2D-I	Exhaustive	1	7	4	11	7	11	11	1	0.112	2.198
	Greedy	1	11	11	7	11	7	4	1	<0.001	2.198
2D-II	Exhaustive	1	4	7	1	11	11	11	7	0.139	1.116
	Greedy	1	4	7	4	11	11	11	1	<0.001	1.234
2D-III	Exhaustive	1	1	11	7	11	7	4	4	0.126	0.461
	Greedy	1	7	11	11	11	7	4	1	<0.001	0.494

TABLE XI

RESULTS FOR THREE-DIMENSIONAL DISTRIBUTIONS WITH THREE CHANNELS

Distribution	Algorithm	Channel assignment: {AP1$_0$, AP2$_0$, AP3$_0$, AP4$_0$, AP5$_0$, AP6$_0$, AP7$_0$, AP8$_0$, AP1$_1$, AP2$_1$, AP3$_1$, AP4$_1$, AP5$_1$, AP6$_1$, AP7$_1$, AP8$_1$}																t_{ex} [s]	F_{tot}
3D-I	Exhaustive	1	6	6	11	6	11	11	1	6	11	11	1	11	1	1	6	240.4	19.553
	Greedy	1	11	1	6	1	6	11	1	6	1	11	11	11	11	6	6	<0.001	19.720
3D-II	Exhaustive	1	1	6	1	11	11	11	6	11	11	1	11	6	6	6	1	239.4	10.497
	Greedy	1	1	6	1	1	1	11	11	6	6	11	6	11	11	1	6	<0.001	10.605
3D-III	Exhaustive	1	1	6	6	6	11	11	11	6	6	11	11	11	1	1	1	238.2	5.347
	Greedy	1	1	11	1	1	6	6	11	6	6	1	11	11	11	11	6	<0.001	5.386

TABLE XII

RESULTS FOR THREE-DIMENSIONAL DISTRIBUTIONS WITH FOUR CHANNELS

Distribution	Algorithm	Channel assignment: {AP1$_0$, AP2$_0$, AP3$_0$, AP4$_0$, AP5$_0$, AP6$_0$, AP7$_0$, AP8$_0$, AP1$_1$, AP2$_1$, AP3$_1$, AP4$_1$, AP5$_1$, AP6$_1$, AP7$_1$, AP8$_1$}																t_{ex} [s]	F_{tot}
3D-I	Exhaustive	1	7	7	11	7	11	11	4	11	1	1	4	1	7	4	11	24003	17.901
	Greedy	1	11	11	4	4	11	7	11	7	1	7	11	11	7	1	4	<0.001	19.710
3D-II	Exhaustive	1	7	11	4	7	11	1	11	7	11	4	11	1	1	7	4	23993	9.304
	Greedy	1	11	4	4	7	7	1	4	7	1	11	11	1	11	7	11	<0.001	9.320
3D-III	Exhaustive	1	4	11	7	7	4	11	7	7	11	4	1	1	11	4	1	24040	4.561
	Greedy	1	11	11	11	7	4	4	7	7	7	4	1	1	11	11	1	<0.001	4.633

TABLE XIII
LOCAL INTERACTION LEVEL FOR THE EXHAUSTIVE ALGORITHM

	Distribution							
	2D-I (3 channels)		2D-I (4 channels)		2D-II (3 channels)		2D-II (4 channels)	
AP no.	F_{c_k}	$F_{k,n} \geq F_{c\_k}$	F_{c_k}	$F_{k,n} \geq F_{c\_k}$	F_{c_k}	$F_{k,n} \geq F_{c\_k}$	F_{c_k}	$F_{k,n} \geq F_{c\_k}$
1	0.30	$F_{1,8} = 0.33$	0.30	$F_{1,8} = 0.33$	0.30	-	0.30	-
2	0.30	$F_{2,3} = 0.33$ $F_{2,5} = 0.33$	0.30	$F_{2,5} = 0.33$	0.11	$F_{2,4} = 0.14$	0.11	$F_{2,3} = 0.11$
3	0.30	$F_{3,5} = 0.33$	0.30	-	0.30	-	0.30	-
4	0.30	$F_{4,6} = 0.33$ $F_{4,7} = 0.33$	0.30	$F_{4,6} = 0.33$ $F_{4,7} = 0.33$	0.30	-	0.30	-
5	0.30	-	0.30	-	0.30	$F_{5,7} = 0.33$	0.30	$F_{5,7} = 0.33$
6	0.30	$F_{6,7} = 0.33$	0.30	$F_{6,7} = 0.33$	0.12	-	0.12	-
7	0.30	-	0.30	-	0.30	-	0.30	-
8	0.30	-	0.30	-	0.30	-	0.30	-

TABLE XIV
LOCAL INTERACTION LEVEL FOR THE GREEDY ALGORITHM

	Distribution							
	2D-I (3 channels)		2D-I (4 channels)		2D-II (3 channels)		2D-II (4 channels)	
AP no.	F_{c_k}	$F_{k,n} \geq F_{c\_k}$	F_{c_k}	$F_{k,n} \geq F_{c\_k}$	F_{c_k}	$F_{k,n} \geq F_{c\_k}$	F_{c_k}	$F_{k,n} \geq F_{c\_k}$
1	0.30	$F_{1,8} = 0.33$	0.30	$F_{1,8} = 0.33$	0.30	-	0.30	-
2	0.30	$F_{2,3} = 0.33$ $F_{2,5} = 0.33$	0.30	$F_{2,5} = 0.33$ $F_{2,5} = 0.33$	0.11	$F_{2,4} = 0.14$	0.11	$F_{2,3} = 0.11$ $F_{2,4} = 0.14$
3	0.30	$F_{3,5} = 0.33$	0.30	$F_{3,5} = 0.33$	0.30	-	0.30	-
4	0.30	$F_{4,6} = 0.33$ $F_{4,7} = 0.33$	0.30	$F_{4,6} = 0.33$	0.30	-	0.30	-
5	0.30	-	0.30	-	0.30	$F_{5,7} = 0.33$	0.30	$F_{5,7} = 0.33$
6	0.30	$F_{6,7} = 0.33$	0.30	-	0.12	-	0.12	-
7	0.30	-	0.30	-	0.30	-	0.30	-
8	0.30	-	0.30	-	0.30	-	0.30	-

VI. CONCLUSIONS

A greedy MST-inspired algorithm for channel assignment in Wi-Fi AP sets was developed and characterized. The algorithm was tested for several sets of APs, both two- and three-dimensional with three and four channels available. The comparison with the exhaustive procedure, which guarantees finding the optimal solution, shows relatively small result deterioration (typically below 10%) and significant advantage in terms of execution time. Especially for sets with more than 10 APs and/or more than three channels available, the exhaustive procedure becomes impractical as its execution time on a low-end CPU will be prohibitively long (even days). Thus, the proposed algorithm can be recommended as a reasonable alternative. Usage of four instead of three channels also turns out to be beneficial for both algorithms; however, the improvement is usually smaller for the greedy one.

Future research should include introduction of three-level AP distributions, consideration of evenness of local interactions, and more realistic model of inter-AP interactions.

REFERENCES

[1] G. Hiertz, R. Denteneer, D. Stibor, P. Lothar, Y. Zang, C. X. Pérea, B. Walke, "The IEEE 802.11 Universe," *IEEE Communications Magazine*, January 2010, pp. 62-70.

[2] S. Chieochan, E. Hossain, J. Diamond, "Channel Assignment Schemes for Infrastructure-Based 802.11 WLANs: A Survey*", IEEE Communications Surveys & Tutorials*, vol. 12, no. 1, 1st Quarter 2010, pp. 124-136.

[3] I. Dolińska, A. Masiukiewicz, G. Rządkowski, "The mathematical model for interference simulation and optimization in 802.11n networks", *in Workshop, Concurrency Specification and Programming* 2013, Warsaw University, 2013, pp. 99-110.

[4] J. Herzen, R. Merz, P. Thiran,, "Distributed Spectrum Assignment for Home WLANs", in *Proc. of IEEE INFOCOM* , 2013, pp. 1573-1581.

[5] P. Gajewski, S. Wszelak, „Optimisation of AC location in WLAN networks with direct finding metod", *Telecommunication Review* no 8-9 2007, pp. 260-265 (in Polish).

[6] Xirrus, (2011), "Wireless solution for High Density Wireless", www.xirrus.com.

[7] I. Dolińska, A. Masiukiewicz, G. Rządkowski, "Channel selection in home 802.11 standard networks", *in Proc. of IEEE Digital Technologies Żylina,* 2014, pp. 58-64.

[8] Y. Bejerano, S. Han, L. Li, "Fairness and Load Balancing in Wireless LANs Using Association Control", *in Proc. of 10th International Conference Mobile Computing and Networking (Mobicom 04*), ACM Press New York 2004, pp. 315-329.

[9] A. Akella, G. Judd, S. Seshan, P. Steenkiste, "Self-Management in Chaotic Wireless Deployments", *in Proc. of 11th International Conference Mobile Computing and Networking (Mobicom 05*), ACM Press New York 2005, pp. 185-199.

[10] B. Alawieh, C. Assi, H. Mouftah, Y. Zhang "Improving Spatial Reuse in Multihop Wireless Networks - A Survey", *IEEE Communications Surveys & Tutorials*,, vol. 11, no. 3, 3rd Quartet 2009, pp. 71-91.

[11] A. Hills, "Large-Scale Wireless LAN Design", *IEEE Communication Magagazine*, vol. 39, no. 11, pp. 98–107, November 2001.

[12] Y. Choi, Y. Lee, K. Kim, "Optimization of AP Placement and Channel Assignment in Wireless LANs", *in Proc. of 27th Annual IEEE Conf. Local Computer Networks*, pp. 831–836, Nov. 2002.

[13] P. Wertz, M. Sauter, F. Landstorfer, G. Wolfle, R. Hoppe, "Automatic Optimization Algorithms for the Planning of Wireless Local Area Networks", *in Proc. IEEE Vehicular Technology Conference,* vol. 4, pp. 3010–3014, Sep. 2004.

[14] X. Ling, K. L. Yeung, "Joint Access Point Placement and Channel Assignment for 802.11 Wireless LANs'", *in Proc. IEEE WCNC'05,* 2005, vol. 3, pp. 1583-1588.

[15] A. Mishra, V. Brik, S. Banerjee, A. Srinivasan, W. Arbaugh, "A Client-driven Approach for Channel Management in Wireless LANs", *in Proc. 25th IEEE International Conference on Computer Communications (INFOCOM'06)*, 2006, vol. 3, pp. 1300-1312.

[16] M. Achanta, "Method and Apparatus for Least Congested Channel Scan for Wireless Access Points", *US Patent No. 20060072602,* Apr. 2006.

[17] R. Akl, A. Arepally, "Dynamic Channel Assignment in IEEE 802.11 networks'," *in Proc. IEEE International Conference on Portable Information Devices (PORTABLE'07)*, 2007, pp. 309-314.

[18] I. Dolińska, A. Masiukiewicz, "Location Ability of 802.11 Access Points", *in Proc. of Conference Information and Digital Technologies,* 2015, Żylina, Slovakia , pp. 233-237.

[19] IEEE Std 802.11k™-2008.

[20] IEEE Std 802.11v™-2011.

[21] R. L. Freeman, "Radio System Design for Telecommunication", *Wiley &Sons* 2007.

[22] I. Dolińska, A. Masiukiewicz, G. Rządkowski, M. Jakubowski, "Algorithms for Channels Assignment in 802.11 Networks", *in Proc. of Conference Information and Digital Technologies* 2016, Rzeszów, Poland, pp .83-89.

[23] D. A. Marcus, "Graph Theory: A Problem Oriented Approach", *Mathematical Association of America,* 2008.

[24] K. Piórczynski, „Tests of RBA algorithm for Chanel assigment in small home Wi-Fi networks", *Diploma* , Vistula University, 2016 (in Polish).

[25] Fuxjäger P., Valerio D., Ricciato F., "The Myth of Non-Overlapping Channels: Interference Measurements in IEEE 802.11", *in Proc. of 4th Annual Conference on Wireless on Demand Network Systems and Service (WONS'07),* 2007, pp. 1-9.