

Analysis of a Compact 4-shaped Annular Ring Ultra Wideband Antenna Using Characteristic Modes

Bhaskara Rao Perli, and A. Maheswara Rao

Abstract—This communication proposes a compact 4-shaped monopole annular ring UWB antenna design. The proposed structure contains multiple radiating strips inside the annular ring, in the form of a 4-shaped and a 50Ω microstrip feed line. A tapered structure with a feed point is chosen to achieve wideband characteristics. The proposed model is printed on a low-priced FR4 substrate with a size of $0.180\lambda_0 \times 0.225\lambda_0$ ($20 \times 25\text{mm}^2$). The proposed model achieves a fractional bandwidth of 133.74% in the 2.7 to 13.6 GHz range with $S_{11} < -10\text{dB}$ and covers the 3.1-10.6 GHz unlicensed band approved by FCC in 2002 and X-band applications. The antenna exhibits stable and Omni-directional radiation patterns in the operating frequency range. The analysis of the proposed monopole antenna using characteristic modes is performed to obtain a physical understanding of the radiation process occurring on the radiating antenna. The modal significance curves and the modal current distributions are used to analyze the radiating antenna using the first six characteristic modes. The measurement and simulation results show a good agreement.

Keywords—characteristic modes; multiple-strips; annular-ring; surface currents; UWB antenna; tapered structure

I. INTRODUCTION

THE Microstrip patch antennas have played a vital role in modern wireless communication with their compelling properties, such as lightweight, easy to fabrication, low profile, and conformable to planar. However, its narrow-bandwidth performance limits its use in UWB technology. In addition, modern wireless communications nowadays usually require small size antennas and greater bandwidth to meet the current requirements of practical applications such as the military, radar, medical imaging, and other wireless communication services. For this reason, substantial research has been increased significantly in the field of bandwidth enhancement and methods of miniaturization for microstrip antennas [1, 2].

The Federal Communications Commission (FCC) selected a frequency range from 3.1 to 10.6 GHz for ultra-wideband applications in 2002, and also increased the demand for the use of this unlicensed frequency band from the industrial and academic domains. UWB technology has also received great attention from academia and industries. A feasible UWB antenna needs to be designed with simple structure, compact size, low cost, low power spectral density, integration capability, near Omni-directional features, and prodigious data rates [3, 4]. Planar monopole antennas are suitable for achieving these characteristics, which is why many researchers

find their interest in the design and analysis of this type of antennas. In previous studies, numerous antennas have been reported for ultra-wideband applications including the entire 3.1 to 10.6 GHz frequency band [5-15].

The UWB antenna with a total dimension of $24.8 \times 30 \times 1.6 \text{ mm}^3$ was proposed by inserting a slot in the ground and radiating structures [5]. The proposed decagonal radiating monopole with a total dimension of $35 \times 35 \times 1.6 \text{ mm}^3$ was truncated at the ground structure in order to observe the UWB band [6]. The V-shaped radiating structure along with a partial ground plane was designed as a UWB monopole antenna with a total size of $24 \times 28 \times 1.6 \text{ mm}^3$ [7]. Multiple-resonators are used to create the longer and shorter current paths for ultra-wideband applications with a total dimension of $66 \times 62 \times 1.59 \text{ mm}^3$ [8]. In [9], the CPW-fed rectangular spiral antenna with a total dimension of $50 \times 40 \times 0.508 \text{ mm}^3$ was designed by tapering the ground plane in the form of coplanar strips but not covering the entire UWB band. A proximity-coupled annular ring antenna with an overall dimension of $44 \times 44 \times 1.42 \text{ mm}^3$ was proposed for UWB application [10]. A flag-shaped monopole antenna with a total dimension of $30 \times 30 \times 1.6 \text{ mm}^3$ consisting of a rectangle strip in asymmetric style and a coplanar waveguide (CPW) fed with finite-ground structure was proposed for ultra-wideband operation [11]. In [12] the designed antenna with a total dimension of $35 \times 24 \times 1.6 \text{ mm}^3$ covers the entire UWB band with a ring-shaped antenna with an upper cutting edge and additional slot, as well as a rectangular slot on the upper middle edge of the partial ground structure. The UWB performance was achieved by the hexagonal patch antenna with a flangeless SMA connector and slots on the truncated ground plane and the radiator with a total dimension of $46 \times 46 \times 1.5 \text{ mm}^3$ [13]. The flower-shaped microstrip patch antenna with coplanar waveguide feeding technique had been proposed for broadband applications but it did not cover the entire UWB band with a total dimension of $28 \times 41.8 \times 1.6 \text{ mm}^3$ [14]. However, Most of the reported literature lacks effective physical insight into the antenna structures used for ultra-wideband operation and are developed based on experimental experience and parametric studies. These antennas are also expected to be small in size for use in portable devices that operate different communication services.

Recently, the use of the CMA in enlightening the physical significance of antennas has become increasingly popular [15]. It is simpler in describing radiating objects of any shape, and provides sufficient information to understand the mechanism of the radiation behind the operation of antenna performance and is attracted by many researchers, primarily for multiple-input and multiple-output (MIMO) antenna [16], wideband and multiband antennas [17,18], and UWB antenna designs [19-21].

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In this paper, using characteristic mode analysis, a compact 4-shaped annular ring planar monopole UWB antenna is proposed. First, CMA is performed on the annular ring in the process of optimizing with a substrate to generate more significant characteristic modes. The surface current distribution of desired modes is analyzing, and then these modes are excited by a simple microstrip line feeding structure and the ground structure. The insertion of strips inside the ring and the tapering the feed line achieves good impedance matching to achieve wide bandwidth. Next, the complete structure is further simulated in a CST multilayer solver to analyze the radiation capabilities of the characteristic modes in the frequency range of interest. The working principle of the proposed model is described from the perspective of the characteristic-mode. Finally, a prototype antenna was made and measured to validate simulation results and the proposed antenna performance compared with existing antenna models.

II. CHARACTERISTIC MODE ANALYSIS

Characteristic mode analysis (CMA) [22] was used to determine the characteristics of the conducting body in terms of modal currents and modal fields. CMA effectively optimizes the size and shape of the conductive body by analyzing the modal current distribution and radiation behavior of each characteristic mode. Characteristic modes are current modes that occur on the surface of a conducting body and are orthogonal to each other. They are evaluated on the surface of the conducting body in order to obtain information about the modal currents (or modal fields) that the structure supports, naturally. The total surface current on the conducting body is defined as a function of the real eigenvalue λ_n , which is related to the n^{th} characteristic mode. The resonant frequencies and radiation bandwidth of the desired characteristic modes are well known by varying λ_n with frequency. Therefore, the modal significance (MS) is defined as a function of λ_n , as

$$MS = \frac{1}{|1+j\lambda_n|}, \text{ a real quantity.} \quad (1)$$

The modal significance is an inherent property of each characteristic mode and quantifies the participation of each mode in the total radiation for a given power supply. MS converts the infinite eigenvalues $[-\infty, +\infty]$ into very finite values $[0, 1]$. If $MS = 1$ for a given mode, the mode is said to be a significant mode, otherwise the mode is said to be an insignificant mode. Any particular mode reaches the maximum value of MS ($MS = 1$) at any frequency, and this frequency is called the resonance frequency of the mode. The modes have higher MS values and contribute more radiation to the total output of the antenna. In most cases, the modal significance is more suitable for analyzing the mechanism of antenna radiation in a wideband of frequencies.

A. Analysis of Proposed Antenna using Characteristic Modes

CMA of the proposed model is performed on a multilayer platform of CST STUDIO SUITE. In this multilayer solver, the radiating element and ground layer are chosen as perfect electric conductors, and the substrate dielectric is selected as the lossless FR-4. CMA is applied to the antenna structure without applying any excitation. R_{in} and R_{out} represent the

inner and outer radii of the basic annular ring geometry as shown in Figure 1. The ratio of outer radius to the inner radius is appropriately selected to produce a more number of significant modes in the operating frequency range. Strips inside the ring and tapering technique are introduced to reduce the resonance frequencies of significant modes.

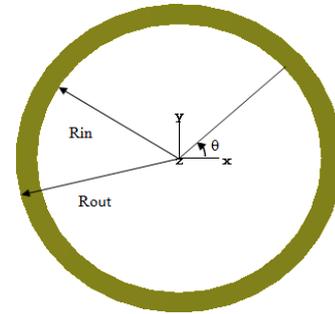


Fig. 1. Geometry of the basic structure of the annular ring ($R_{in} = 7$ mm, $R_{out} = 8$ mm)

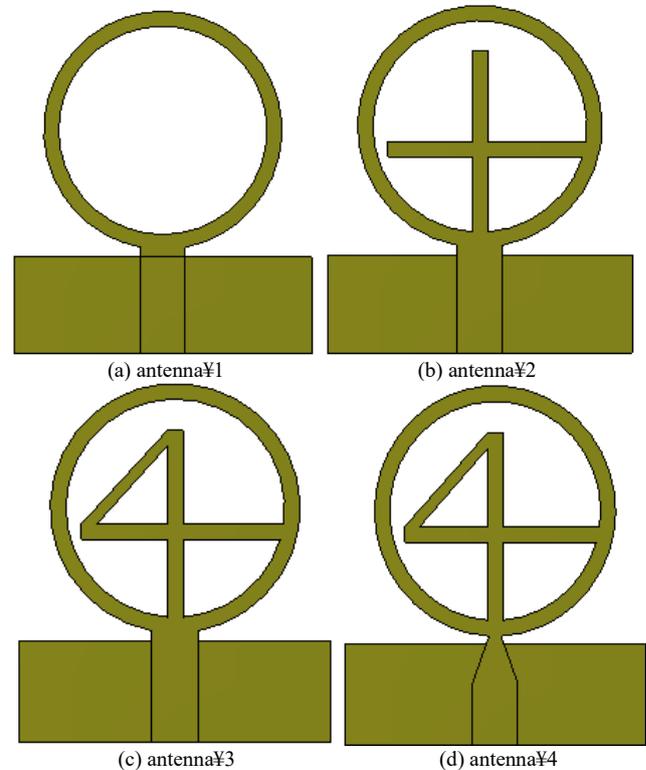


Fig. 2. Design steps of proposed monopole antenna.

Modal significance provides the resonant frequency and bandwidth of each mode with MS equal to 1. Figure 3a–d illustrates the MS curves of the first six characteristic modes for all the monopole antennas depicted in Figure 2. It can be noted that due to the non-resonating behavior of mode 3, which exhibits inductive nature over the operating frequency. The resonant frequencies of other modes associated with the characteristic currents appear at certain frequencies, as shown in Table I.

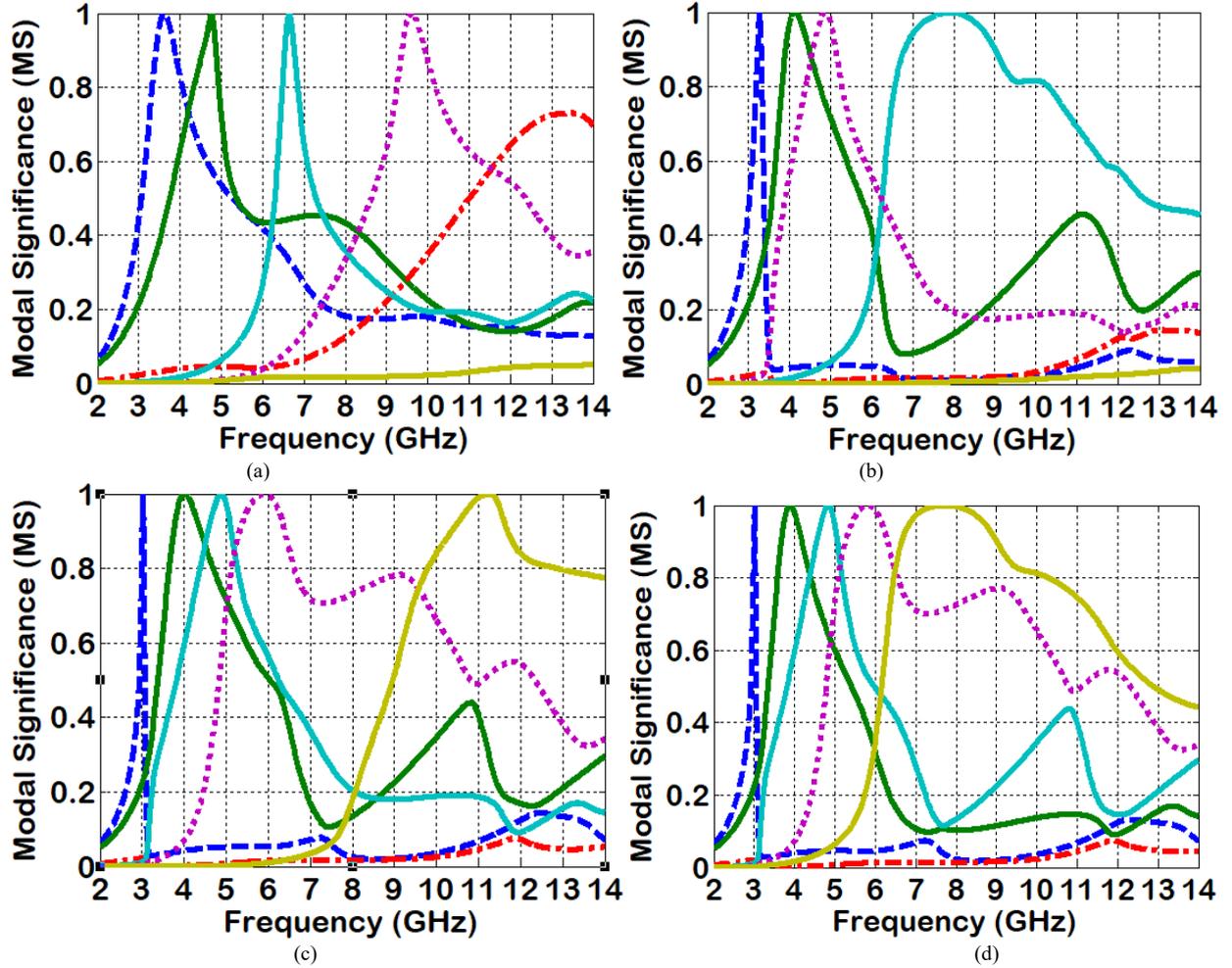


Fig. 3. Modal significance curves of (a) antenna#1 (b) antenna#2 (c) antenna#3 and (d) antenna#4

TABLE I
RESONANT FREQUENCIES OF SIGNIFICANT MODES AS A FUNCTION OF
FREQUENCY (GHz) FOR VARIOUS ANTENNAS

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
antenna#1	3.65	4.75	-	6.64	9.6	-
antenna#2	3.3	4.2	-	8.0	4.9	-
antenna#3	3.0	4.0	-	4.9	5.9	11.3
antenna#4	3.0	3.9	-	4.9	5.8	7.7

It can be observed that the lower resonating frequencies mainly depends on modes 1 and 2. Since the monopole antenna is modified step by step from antenna#1 to the proposed antenna to optimally match the input impedance of the antennas, the resonant frequencies of these modes 1, 2 changes to lower values. For the antenna#4, this explains the reason for the improved performance at the lower frequencies. As for the antenna#2, CMA depicted that mode 5 resonates at 4.9 GHz, which is less than the resonance frequency of mode 4. In addition, for antenna#3 and antenna#4, new mode i.e. namely mode 6 is generated at a higher frequency. When the resonance frequency of mode 6 shifts from 11.3 to 7.7 GHz in the case of the proposed antenna, it takes part in achieving UWB characteristics, because the resonance frequencies of

other modes are remaining almost the same as those observed in Table I. Therefore, for the proposed model, the lower frequency band is controlled by mode 1 and mode 2, the middle frequency part by modes 4 and 5, and the last frequency part by mode 6. However, it is not easy to distinguish the effects of each mode under these higher harmonics, because multiple resonant modes will result in higher resonant frequencies.

The characteristic currents on the conducting surface depend only on the shape, size, and operating frequency, but not on any particular source or excitation. The total current on the surface of the antenna structure can be computed as a sum of the orthogonal currents of the conducting body. In the same way, the radiation field pattern can also be estimated based on characteristic fields that radiates independently. Therefore, the characteristic mode analysis not only demonstrates the resonance behavior of the antenna structure but also gains a physical intuition about the radiation and operation of the antenna structure.

Fig. 4 presents the distribution of the surface currents of the first six characteristic modes of the proposed model. Except for mode 3, all mode currents will cause antenna resonances. Mode 3 consists of non-resonant current behavior, which forms a circular current distribution and stores the magnetic-

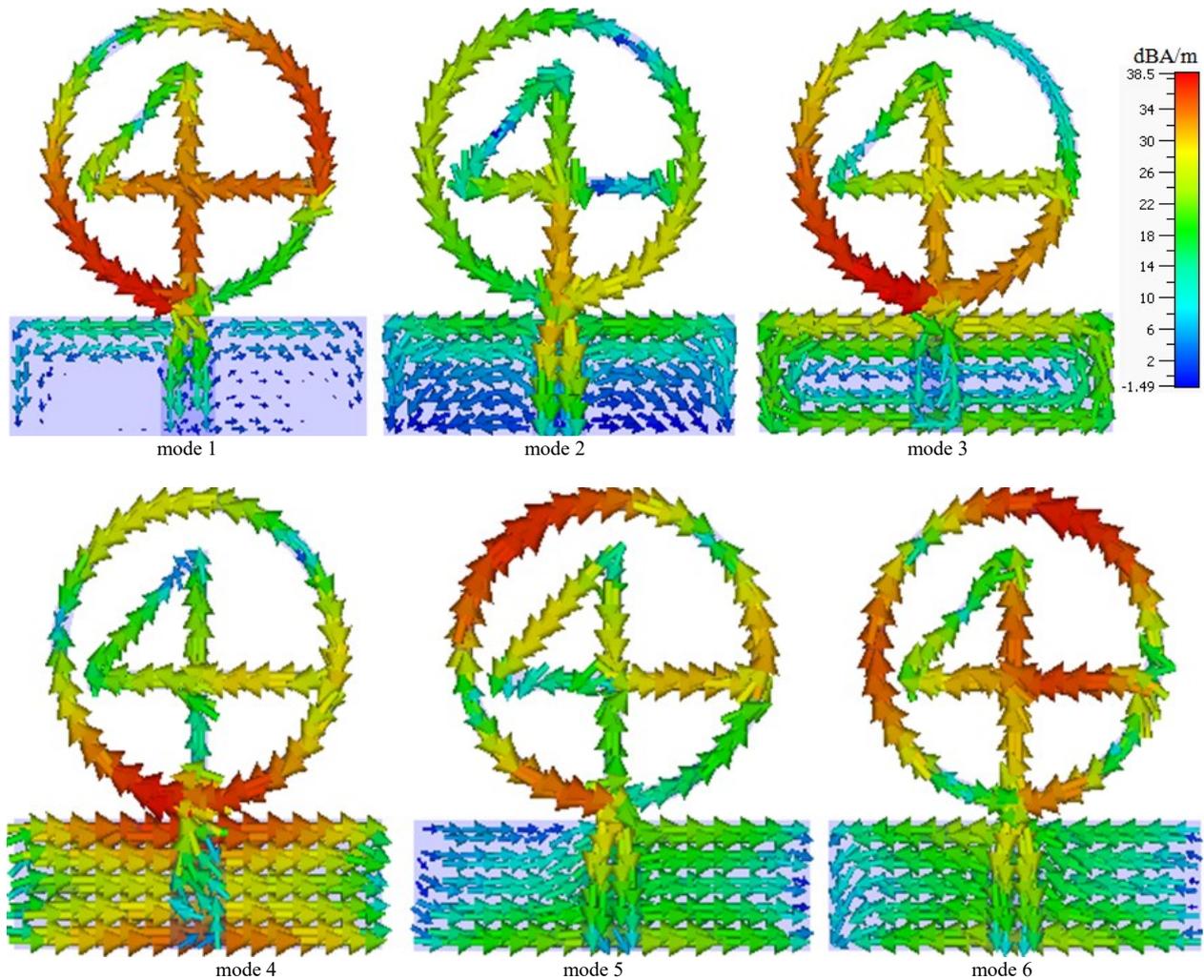


Fig. 4. Distribution of surface currents for modes 1-6 at their resonant frequencies of the proposed model

energy of the antenna due to its inductive nature. It should be noted that these surface current distributions do not depend on the feed configuration but on the shape and size of the antenna. An optimum feeding location will excite the significant modes and show good antenna performance.

In the proposed model, for mode 1, the most currents flow in the annular ring and cross-shaped strip, while for mode 2, the more current flow in the annular ring with a very small current in the 4-shaped strip. However, for modes 1 and 2, the rectangular ground has very small current flows. For mode 4, the strong currents are distributed over the annular ring, cross-shaped strip, and the rectangular ground. On the other hand, for modes 5 and 6, multiple current nulls are identified with the distribution of strong currents on the annular ring and 4-shaped strip and less current on the rectangular grounds. Because the mode currents are independent of antenna excitation, the proper positioning of the feed point on the antenna structure is very important in order to excite certain characteristic modes. In Figure 4, it is also observed that the tapered microstrip line serves as a feed structure and provides consistently strong current distribution and excites the desired mode current distributions in the proposed model.

The 3D modal radiation patterns of most significant modes of a proposed model at their mode resonance frequencies such as 3, 3.9, 4.9, 5.8, and 7.7 GHz are presented in Figure 5. The modal fields show the Omni-directional radiation properties in the E (YZ) and H (XZ) planes. Modes 1 and 2 are more significant and display good patterns of radiation at lower frequencies. Modes 4 and 5 are more significant and show better radiation patterns in mid-frequencies. Mode 6 also shows a good pattern of radiation at higher frequencies. But for mode 4, the radiation is slightly reduced in the Z-direction. Mode 3 is ignored due to null in the broadside direction. However, the maximum radiation of the proposed model is still relatively high in the Z- direction.

III. DESIGN OF A ULTRA-WIDEBAND ANTENNA AND EVOLUTION PROCESS

The geometry of the basic annular ring antenna and proposed an ultra-wideband 4-shaped annular ring monopole antenna as depicted in Figure 6. The proposed model is excited by connecting the waveguide's power port at one end of the 50Ω microstrip line and another end connected to the radiant patch.

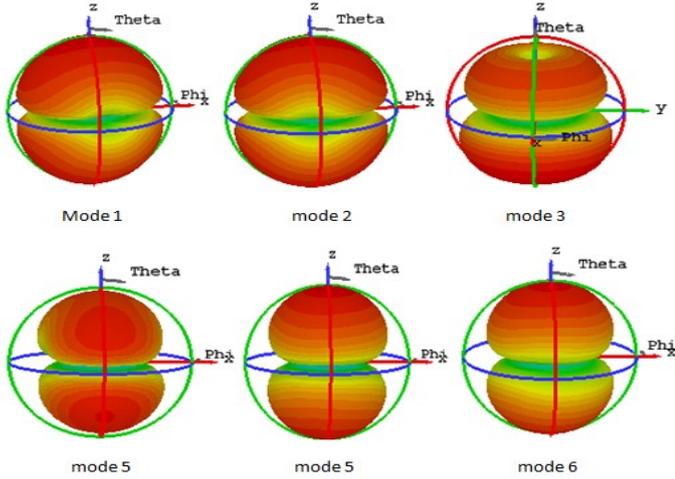


Fig. 5. The 3D modal fields of the 1-6 characteristic modes of the proposed model at their resonant frequencies

The substrate chosen to fabricate the antenna is FR4 with $h = 1.6$ mm, $\tan \delta = 0.02$, and $\epsilon_r = 4.3$. The proposed model has a substrate of dimensions ($S_w \times S_L$) with the partial ground plane of ($G_w \times G_L$). The microstrip feed line has a size of ($F_w \times F_L$) with tapered feed width F_t . Multiple strips are introduced inside the radiating patch to decrease the resonance frequency and tapering the feed line near the radiating patch takes an imperative task to broaden the bandwidth of the proposed model from 2.7 to 13.6 GHz. Great portrayals as far as compact size, wideband and radiation properties are the benefits of an annular ring-shaped antenna. Table II shows the proposed model dimensions.

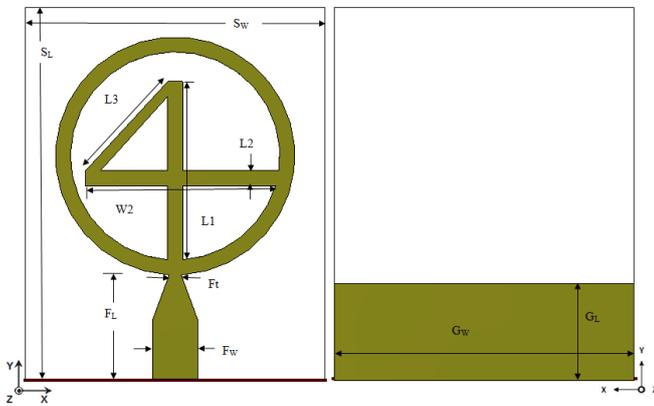


Fig. 6. Proposed ultra-wideband monopole 4-shaped annular ring antenna

TABLE II
THE DIMENSIONS OF THE PROPOSED MODEL

Parameter	Value [mm]	Parameter	Value [mm]
S_w	20	G_L	6.5
S_L	25	G_w	20
F_w	03	L_1	13
F_L	7.5	L_2	01
F_t	0.8	L_3	06
R_{in}	07	W_2	13
R_{out}	08		

IV: RESULTS AND DISCUSSION

The measurement of the electrical properties of the proposed model is determined by the Anritsu MS2037C/2 network analyzer. Figure 7 shows the prototype of the fabricated antenna.

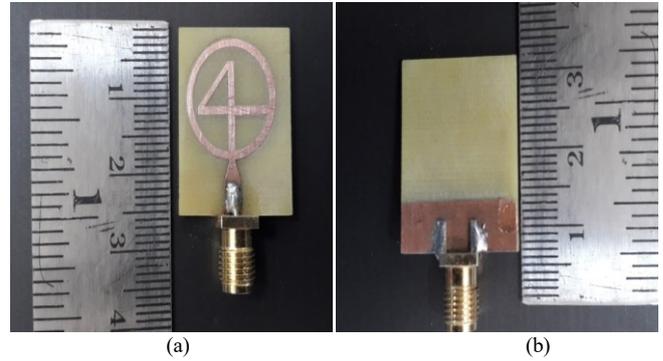
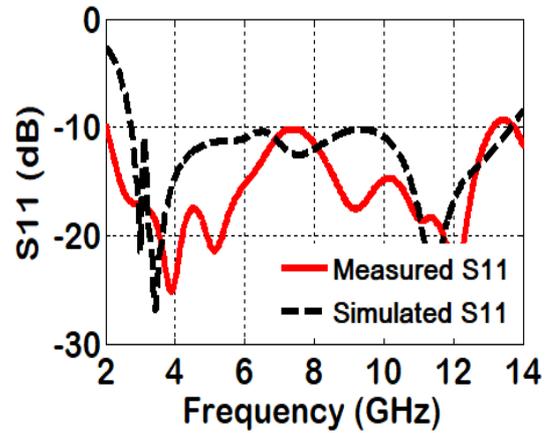
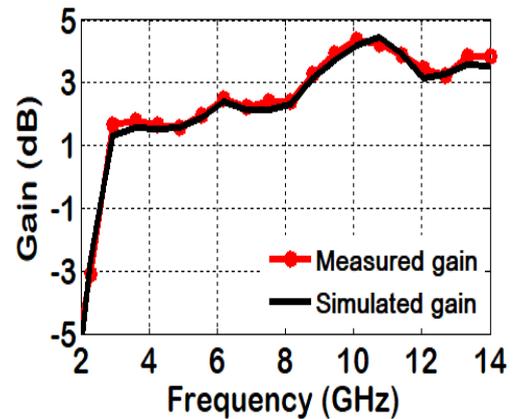


Fig. 7. Photograph of (a) & (b) the top and bottom views of the fabricated antenna.



(a)



(b)

Fig. 8. Proposed antenna measured and simulated (a) S_{11} plot, (b) gain plot

Figure 8(a) illustrates that the measured and simulated return loss bandwidths of the proposed model are 130.33 % (2.7 GHz–12.8 GHz) and 146.66 % (2 GHz–13 GHz). From the return loss curves, it can be observed that modes 1 and 2 govern the lower resonance frequencies and the higher resonance frequencies are controlled by the higher order-

modes 5 and 6. The broadside gain of the proposed model as a function of frequency in GHz is presented in Figure 6(b). The simulated gain of antenna is in the range of 1.7 dBi (3.1 GHz) to 4.45 (10.6 GHz) dBi, which is a nearly stable gain over the operational UWB band. At higher frequencies, the gain remains above 3 dBi. The measured and simulated outputs show a little discrepancy due to the fabrication tolerances and the soldering of the connector to the feed line. As shown in Table 2, the dimensions and bandwidth of the proposed work are compared with the performance of existing antennas provided in references [5-14, 19-21]. The proposed model is compact in comparison with [5-14, 19-21] for UWB operation. Therefore, the proposed model is the finest in terms of compactness and can maintain a constant gain in the working UWB frequency band.

TABLE III
PERFORMANCE COMPARISON BETWEEN PROPOSED WORK AND EXISTING ANTENNA WORKS

Reference	Size (mm ²)	Area (mm ²)	Bandwidth, GHz	% BW	Impedance BW
5	30 × 51	1530	3-12.6	123.1	9.6:1
6	35 × 35	1225	3-12.6	123	9.6:1
7	24 × 28	672	3.15-13.2	124	10.15:1
8	66 × 62	4092	2.4-12	100	9.6:1
9	50 × 40	2000	3.5-11	103.45	7.5:1
10	44 × 44	1936	2.8-12.3	125.83	9.5:1
11	30 × 30	900	3.1-12	117.88	8.9:1
12	35 × 24	840	3.088– 12.497	120.74	9.409:1
13	46 × 46	2116	2.3 to 10.6	129	8.3:1
14	28 × 41.8	1170.4	2.2 to 10.1	131.71	7.9:1
19	28 × 35	980	3 to 11	110	9.0:1
20	30 × 30	900	2.6-12	128.77	9.4:1
21	23.1 × 39.5	912.45	2 -12	142.86	10:1
This work	20 × 25	500	2.7 to 13.6	133.74	10.9:1

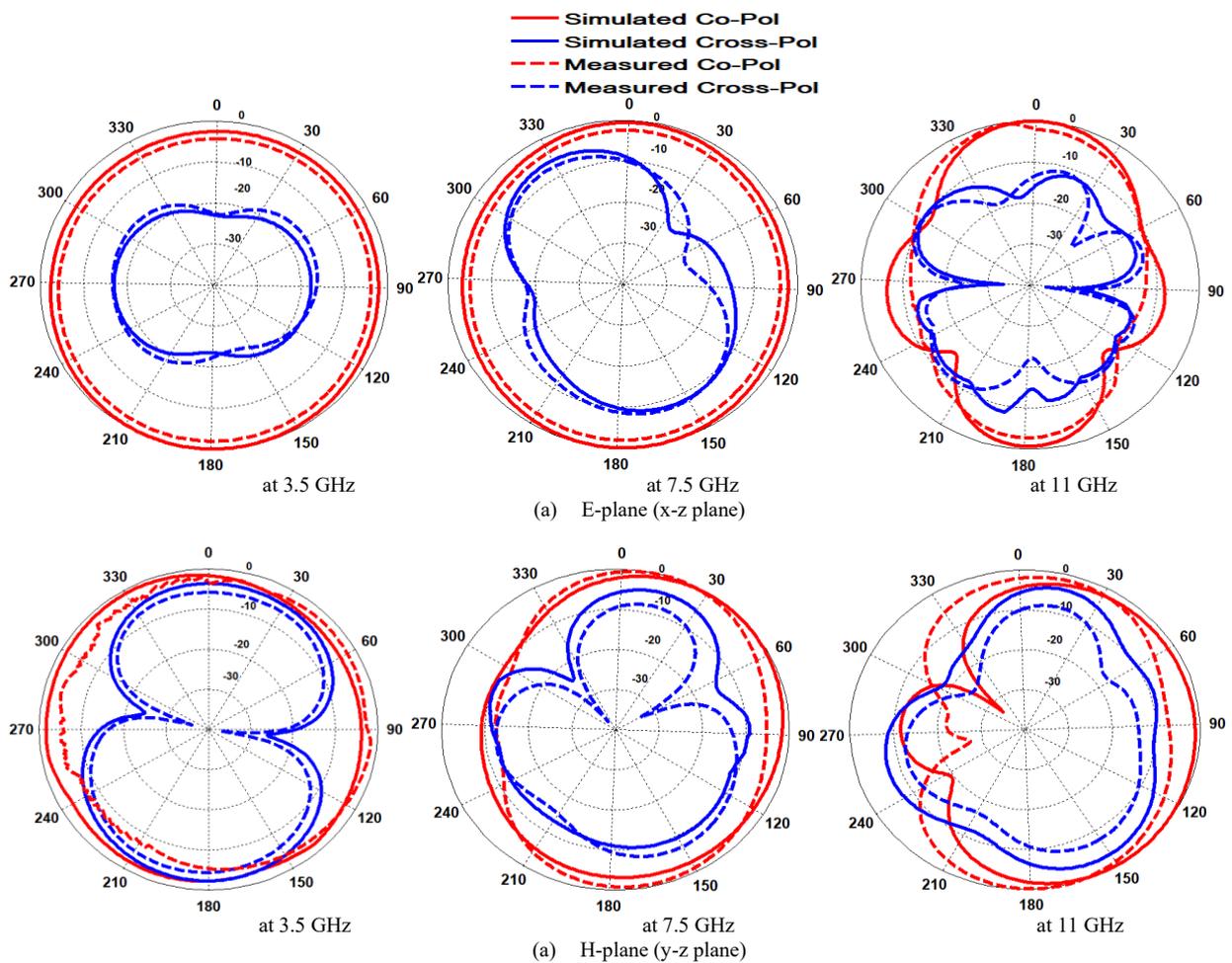


Fig. 9. The 2D radiation patterns of the proposed model at different resonant frequencies.

Figure 9 shows the simulated and measured radiation patterns of the proposed model in the E- and H-planes at three different frequencies of 3.5, 7.5, and 11 GHz. It can be seen that at frequencies of 3.5 GHz and 7.5 GHz, the radiation patterns of the UWB antenna are omnidirectional both in the E-plane and H-plane. At 11 GHz, the radiation pattern shows almost omnidirectional characteristics. Due to the presence of higher-order modes, the radiation pattern in the E-plane and H-plane

is distorted at a higher frequency of 11 GHz. The proposed model exhibits stable radiation patterns in the H-plane and E-plane over the operating frequency range.

CONCLUSION

A compact 4-shaped annular ring antenna has been proposed for both ultra-wideband and the X-band. This antenna has a small dimension of $0.180\lambda_0 \times 0.225\lambda_0$ (20×25 mm²), where

λ_0 is a wavelength related to a lower cut-off frequency. The multiple-strips and tapered technique improve the fractional impedance bandwidth by up to 133.74% and range from 2.7 GHz to 13.6 GHz. The characteristic mode analysis is adopted to gain deep insight into the physics behind the improved performance. The radiation performance of each characteristic mode in the operating frequency range was analyzed with the help of modal significance curves, mode currents, and radiation patterns. The performance of the proposed model in terms of frequency and time domains makes it well suitable for ultra-wideband applications.

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